An extension of the earthquake flow

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Content

- Recall measured laminations, earthquakes,
- Extension to "landslides",
- Underlying AdS geometry.

Joint work with Francesco Bonsante and Gabriele Mondello.

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Measured laminations

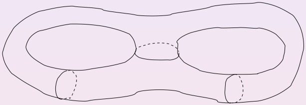
$\mathcal{WM}$ = { weighted multicurves on S } : set of disjoint simple closed curves, each with a positive weight.

$\mathcal{WM}$ is infinite : simple closed curves on S can wrap around a lot.

Let $(c_i, l_i)_{i=1,\cdots,n} \in \mathcal{WM}$, the c_i form a *lamination* and the l_i define a *transverse measure* : gives a total weight to γ, transverse to the c_i.

This gives a topology to $\mathcal{WM}$.

The completion of $\mathcal{WM}$ is the space of *measured laminations* $\mathcal{ML}$.



Measured laminations can be pretty complicated.

- $\mathcal{ML} \simeq \mathbb{R}^{6g-6}$.
- $\partial\mathcal{T} \simeq \mathcal{ML}/\mathbb{R}_{>0}$ (Thurston).
- $\mathcal{T} \times \mathcal{ML} \simeq T^*\mathcal{T}$.

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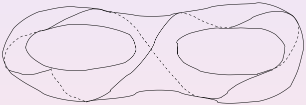

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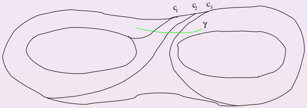

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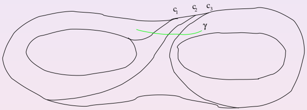

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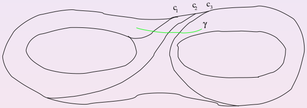

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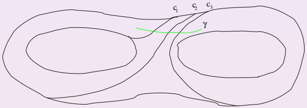

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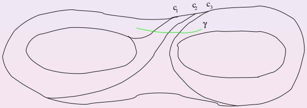

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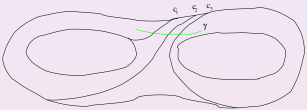

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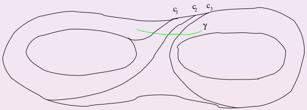

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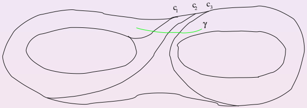

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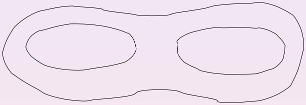
Earthquakes

Start with a hyperbolic surface.

If $w \in \mathcal{ML}$ is a weighted curve and $h \in \mathcal{T}$, $E_w(h)$ is obtained by realizing w as a geodesic in h, cutting S open along w, turning the left-hand side by the weight, and gluing back.

Defines a homeomorphism

$$E_w : \mathcal{T} \to \mathcal{T} \ .$$



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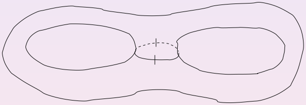

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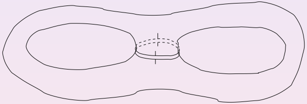

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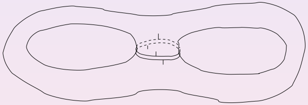

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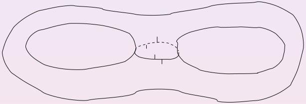

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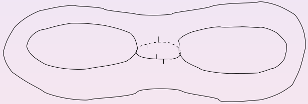

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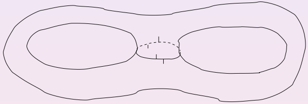

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Some key properties

1. Earthquakes define a *flow* on $\mathcal{T}\times\mathcal{ML}$: $E_{s\lambda}\circ E_{t\lambda} = E_{(s+t)\lambda}$.
2. **Earthquake Thm** (Thurston, Kerckhoff) : $\forall h, h' \in \mathcal{T}, \exists ! \lambda \in \mathcal{ML}, E_\lambda(h) = h'$.
3. Complex earthquakes (McMullen) : for $(h,\lambda)\in\mathcal{T}\times\mathcal{ML}$, the map $t\mapsto E_{t\lambda}(h)$ extends to a holomorphic map $\mathbb{H}\to\mathcal{T}$.
4. $E_{(t+is)\lambda} = gr_{s\lambda}\circ E_{t\lambda}$, where $gr_\lambda : \mathcal{T}\to\mathcal{T}$ is the grafting map.

$gr = \pi\circ Gr : \mathcal{ML}\times\mathcal{T}\overset{Gr}{\to}\mathcal{CP}\overset{\pi}{\to}\mathcal{T}$, and $t+is\mapsto Gr_{s\lambda}\circ E_{t\lambda}$ is holomorphic from $\mathbb{H}$ to $\mathcal{CP}$.

5. $Gr : \mathcal{ML}\times\mathcal{T}\to\mathcal{CP}$ is a homeo (Thurston).

Simple proof of Earthquake Thm by Mess (1990) based on AdS geometry.

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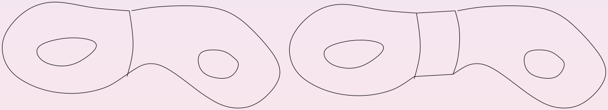

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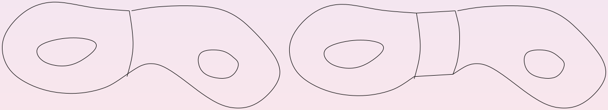

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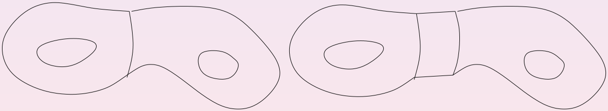

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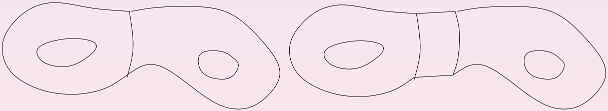
3. Complex earthquakes (McMullen) : for $(h, \lambda) \in \mathcal{T} \times \mathcal{ML}$, the map $t \mapsto E_{t\lambda}(h)$ extends to a holomorphic map $\mathbb{H} \to \mathcal{T}$.
4. $E_{(t+is)\lambda} = gr_{s\lambda} \circ E_{t\lambda}$, where $gr_\lambda : \mathcal{T} \to \mathcal{T}$ is the grafting map.



$gr = \pi \circ Gr : \mathcal{ML} \times \mathcal{T} \xrightarrow{Gr} \mathcal{CP} \xrightarrow{\pi} \mathcal{T}$, and $t + is \mapsto Gr_{s\lambda} \circ E_{t\lambda}$ is holomorphic from $\mathbb{H}$ to $\mathcal{CP}$.

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Simple proof of Earthquake Thm by Mess (1990) based on AdS geometry.

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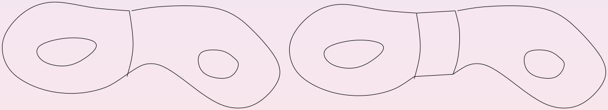

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The landslide flow

Recall that

$$\begin{aligned} E: \quad \mathcal{T}\times\mathcal{ML}\times\mathbb{R} &\to \mathcal{T}\times\mathcal{ML} \\ (h,\lambda,t) &\mapsto (E_{t\lambda}(h),\lambda) \end{aligned}$$

is a flow ($\mathbb{R}$-action).
We'll define "landslides"

$$\begin{aligned} L: \quad \mathcal{T}\times\mathcal{T}\times S^1 &\to \mathcal{T}\times\mathcal{T} \\ (h,h^*,e^{i\theta}) &\mapsto L_{e^{i\theta}}(h,h^*) \end{aligned}$$

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Minimal Lagrangian maps

First possible definition : by minimal Lagrangian maps.

Def : a diffeomorphism u between two hyperbolic surfaces (S, h) and (S, h^*) is minimal Lagrangian if it is area-preserving and its graph is minimal.

Then $u : w \circ v^{-1}$, where $v : (S, c) \rightarrow (S, h)$ and $w : (S, c) \rightarrow (S, h^*)$ are harmonic maps with opposite Hopf differential$q, -q$.

Example : if S is a constant curvature surface in a constant curvature 3-manifold, then $Id : (S, I) \rightarrow (S, III)$ is minimal Lagrangian.

Thm (Schoen, Labourie 1992) : there is a unique minimal Lagrangian diffeo isotopic to the identity between two hyperbolic metrics on S.

Def : $L_{e^{i\theta}}(h, h^*) = (h_\theta, h^*_\theta)$, where the harmonic map $v_\theta : (S, c) \rightarrow (S, h_\theta)$ has Hopf differential $e^{i\theta} q$ (and similarly for w_θ).

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$$AdS_3 = \{x \in \mathbb{R}^{2,2} \mid \langle x, x \rangle = -1\} .$$

Constant curvature -1, $\pi_1(AdS_3) = \mathbb{Z}$.

- Conformal model, in a cylinder.
- Projective model, in a quadric.
- Space-like, time-like, light-like directions. Time-like geodesics are closed of length 2π.
- Totally geodesic space-like planes $\simeq H^2$.
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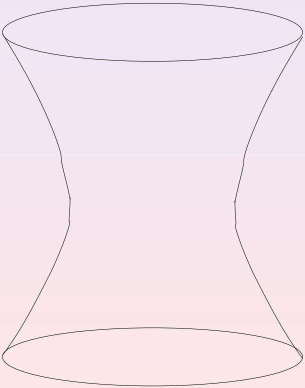

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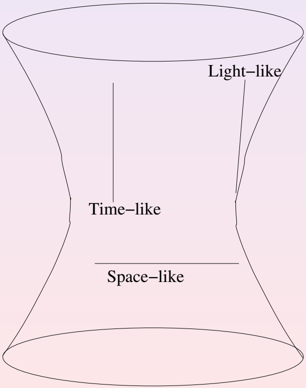

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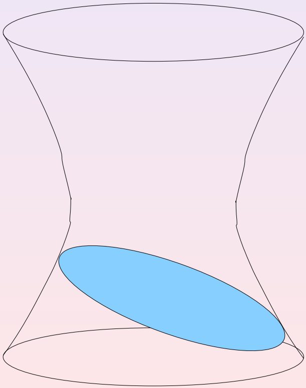

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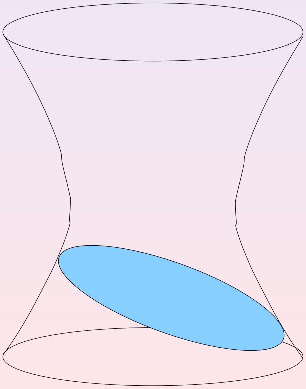

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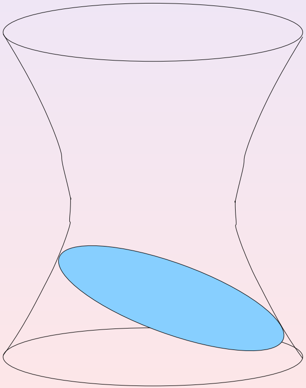

AdS_3 as a Lorentz analog of S^3

Recall : $S^3 = SU(2) \simeq SO(3)$, and $Isom(S^3) = O(4) \simeq O(3) \times O(3)$.
$AdS_3 = PSL(2, \mathbb{R})$ with its Killing metric. Left and right actions of $PSL(2, \mathbb{R})$, identifies $Isom_0(AdS_3) = PSL(2, \mathbb{R}) \times PSL(2, \mathbb{R})$ (up to index 2).

Geometrically :

- $\partial_\infty AdS_3$ is foliated by 2 families of lines.
- Thus $\partial_\infty AdS_3 \simeq \mathbb{R}P^1 \times \mathbb{R}P^1$,
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Recall : $S^3 = SU(2) \simeq SO(3)$, and $Isom(S^3) = O(4) \simeq O(3) \times O(3)$.
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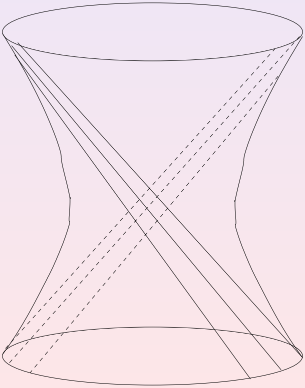

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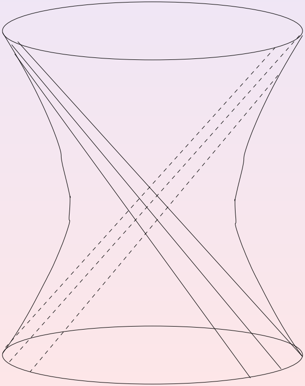

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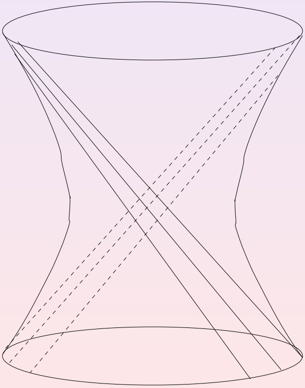

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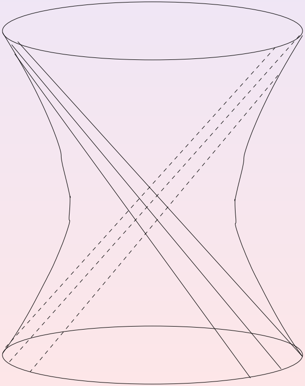

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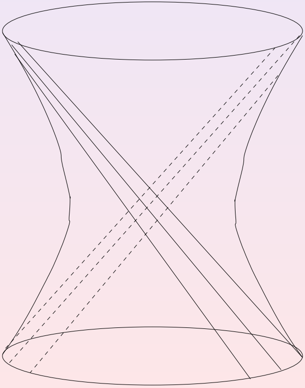

Globally hyperbolic AdS manifolds

Def. an AdS mfld M is *maximal globally hyperbolic* if

- it contains a closed, space-like surface S,
- any inextendible time-like curve intersects S exactly once,
- it is maximal (for inclusion) under those properties.

Then $M \simeq S \times \mathbb{R}$, and $M = \Omega/\rho(\pi_1 S)$, where $\Omega \subset AdS_3$.
GH AdS mflds are strongly reminiscent of quasifuchsian hyperbolic mflds, but in a way more relevant to Teichmüller theory (Mess).

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A Bers-type parametrization

Given a GHMC AdS mfld M, $\rho : \Gamma \to SO(2,2) \simeq PSL(2,\mathbb{R}) \times PSL(2,\mathbb{R})$.
So, $(\rho_L, \rho_R) : \Gamma \to PSL(2,\mathbb{R})$.
Thm (Mess).

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Landslides and 3d geometry

Def : let $h, h^* \in \mathcal{T}$ and let $e^{i\theta} \in S^1$. There is a unique equivariant embedding of S in AdS_3 with $I = 1/\cos^2(\theta/2)h, III = 1/\sin^2(\theta/2)h^*$. S is contained in a unique GH AdS 3-manifold. $L_{e^{i\theta}}(h, h^*) = (h_\theta, h^*_\theta)$ where h_θ is the left representation of M, and $h^*_\theta = h_{\theta+\pi}$.

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A word on the proofs

0. Limit to earthquakes : if $t_n h_n^* \to \lambda$, then $L^1(h, h_n^*, e^{i\theta_n}) \to E_\lambda(h)$.

Technical issues but main idea is convergence of K-surfaces to the boundary of the convex core of a GH AdS manifold when $K \to -1$. A statement of independent interest is hidden.
Thm : Suppose $t_n h_n^* \to \lambda$ (length spectrum), and suppose that the identity between (S, h) and (S, h_n^*) is minimal Lagrangian. Then for any segment $\gamma \subset S$, with endpoints $\notin supp(\lambda)$, $L_{t_n h_n^*}(\gamma) \to i(\gamma, \lambda)$.

1. L is a flow (S^1-action) : $L_{e^{i\theta}} \circ L_{e^{i\theta'}} = L_{e^{i(\theta+\theta')}}$.

Long computation, but no geometric explanation (yet).

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Proof uses a recent result by Barbot, Béguin, Zeghib, on existence and uniqueness of foliation by K-surfaces of GH AdS manifolds.

3. Complex extension : $L^1_{\cdot}(h, h^*) : S^1 \to \mathcal{T}$ extends to a holomorphic map $D \to \mathcal{T}$.

Long computation/argument, however a nice geometric argument seems possible based on surfaces in $PSL(2, \mathbb{C})$.

4. "Smooth grafting" : for $r \in (0,1)$, $L^1_r : \mathcal{T} \times \mathcal{T} \to \mathcal{T}$ is a smooth version of grafting, sgr_r. $sgr_r = \pi \circ SGr_r$, where $SGr_r : \mathcal{T} \times \mathcal{T} \to \mathcal{CP}$, and $s + it \to SGr_{e^{-s}} \circ L_{e^{it}}(h, h^*)$ is holomorphic on $\mathbb{H}$.

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Proof uses a recent result by Barbot, Béguin, Zeghib, on existence and uniqueness of foliation by K-surfaces of GH AdS manifolds.

3. Complex extension : $L^1_.(h, h^*) : S^1 \to \mathcal{T}$ extends to a holomorphic map $D \to \mathcal{T}$.

Long computation/argument, however a nice geometric argument seems possible based on surfaces in $PSL(2, \mathbb{C})$.

4. "Smooth grafting" : for $r \in (0, 1)$, $L^1_r : \mathcal{T} \times \mathcal{T} \to \mathcal{T}$ is a smooth version of grafting, sgr_r. $sgr_r = \pi \circ SGr_r$, where $SGr_r : \mathcal{T} \times \mathcal{T} \to \mathcal{CP}$, and $s + it \to SGr_{e^{-s}} \circ L_{e^{it}}(h, h^*)$ is holomorphic on $\mathbb{H}$.

Based on hyperbolic geometry.

5. $SGr_r : \mathcal{T} \times \mathcal{T} \to \mathcal{CP}$ is a homeomorphism.

Follows from older result of Labourie on constant curvature surfaces in hyperbolic ends.

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There is a number of natural questions :

- Is the landslide flow a Hamiltonian flow ?
- Extension of result of Scannell-Wolf on grafting being homeomorphism ?

Some of those questions have simple translations in terms of 3d geometry.

Thanks for your attention !

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