Aspects of Hyperbolicity in Geometry, Topology and Dynamics
Problem Session, Warwick, 27th July 2011
Chair: Yair Minsky
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These are informal notes from our problem session and discussion. Thanks to all those who participated. We (Saul and Brian) welcome comments or news on any of these problems.

(1) (Caroline Series) What is Teichmüller theory good for? Give examples of the "Impact" of geometry and topology on the real world.
Comments:
(Kasra Rafi) A talk by Milnor about the phase space of some species being hyperbolic. Related to work of Shahshahani?
(Vaibhav Gadre) Work by Jean-Luc Thiffeault on applications of pseudoanosov dynamics to fluid mixing.
Also work by Boyland, Aref, Stremler, Kobayashi etc.
(Dorothy Buck) DNA-protein interaction
(Jeff Brock) Mumford using Weil-Petersson metric on shape space to understand computer vision.
(Dorothy Buck) Following on, related ideas used are used for imaging.
(Various participants) Applications of circle packing. Stochastic Loewner evolution. Conformal invariance of percolation, etc.
(Caroline Series) Is there some kind to theory about how leaf shapes develop and bend into 3-space? Particularly negatively curved like kale etc.

(2) (Kasra Rafi) Given three projective laminations, λ, μ, σ on a closed surface Σ, when does σ lie in the limit set $\partial D(\lambda, \mu)$, where $D(\lambda, \mu)$ is the complex Teichmüller disc with $\sigma \in \partial D(\lambda, \mu)$?
Describe which simple curves on Σ are short in some structure in $D(\lambda, \mu)$.
(Jeff Brock) What does the limit set of $D(\lambda, \mu)$ in the Thurston compactification look like?

(3) (Brian Bowditch) Is it possible to embed a Cantor set in the 3-sphere in such a way that its complement admits a complete hyperbolic structure? (One could also ask for a negatively curved metric etc.)

(4) (Yair Minsky) Let M be a 3-manifold with no essential spheres or tori. Suppose that every cover of M with finitely generated fundamental group has a hyperbolic structure. Does M admit a hyperbolic structure?

(5) (Jeff Brock) Let $||\phi||_{\text{pants}}$ and $||\phi||_{\text{WP}}$ be the minimal translation distances of a pseudoanosov on the pants complex and Weil-Petersson metric respectively. Brock shows that

there is some $K > 0$ such that

$$\frac{1}{K} \leq \frac{||\phi||_{\mathrm{pants}}}{||\phi||_{\mathrm{WP}}} \leq K.$$

How does K depend on the complexity of the surface?

Can it be chosen independently of the surface?

One can ask similar questions involving the volume of the hyperbolic 3-manifold with monodromy ϕ.

(6) (Brian Bowditch) Can one compute stable lengths of pseudoanosovs acting on the curve complex?

Remark: It is known that these are uniformly rational. One wants a computable bound on the denominators in terms of of complexity of the surface.

(7) (Vaibhav Gadre) (Following on from (6)) Note that the stable length spectrum for the mapping class group acting on the curve complex is discrete, but with infinite multiplicity. Which stable length is smallest with infinite multiplicity? Is this the same as the smallest stable length?

(8) (Jeff Brock) Can one compute the Weil-Petersson length of anything at all?

Which is the shortest W-P geodesic on any given surface?

(9) (Kasra Rafi) Does there exist a sequence, ϵ_g, with $\epsilon_g \to \infty$ as $g \to \infty$, such that one can find a pseudoanosov ψ_g on the surface of genus g whose axis lies in the ϵ_g-thick part of Teichmüller space with the Weil-Petersson metric?

Probably, yes, using covers. But one can refine the question: for example, to place an upper bound on the lengths of the axes of the ψ_g.

(Vlad Markovic) Can one do this with $\epsilon_g = k \log(g)$?

(10) (Brian Bowditch) Can one find a properly embedded plane in Teichmüller space with the Weil-Petersson metric such that the induced metric is negatively curved, and such that the total curvature (integral of the curvature) is arbitrarily small?

(11) (Vlad Markovic) Is every isometric embedding of the hyperbolic plane into Teichmüller space with Teichmüller metric a Teichmüller disc?

(Brian Bowditch) Following on: One could ask the same question, replacing "the hyperbolic plane" with "any riemannian disc".

(12) (Brian Bowditch) Does there exist a left-invariant Finsler metric on the Heisenberg group which is non-positively curved in the sense of Busemann (i.e. a geodesic space where the distance function is convex on pairs of geodesic segments)?

Remark: This is not possible for CAT(0). This would have to be riemannian, and hence the euclidean plane. Contradiction.

(13) (Vaibhav Gadre) Which algebraic integers arise as dilation factors of pseudoanosovs? The degree is bounded in terms of the genus, but are there other constraints?

(14) (Caroline Series) What to the hairy bits poking out of the Maskit slice in the conference poster really look like?