Dehn filling and knot complements that don’t irregularly cover

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GaTO 2020

Setting: cusped hyperbolic 3-manifolds

(orientable, connected, finite volume)

↪ torus cusps

Q: What hyperbolic 3-mflds can cover / be covered by a knot complement?

for example...

Q': Can a hyperbolic knot complement cover another knot complement?

Yes! Berge knots

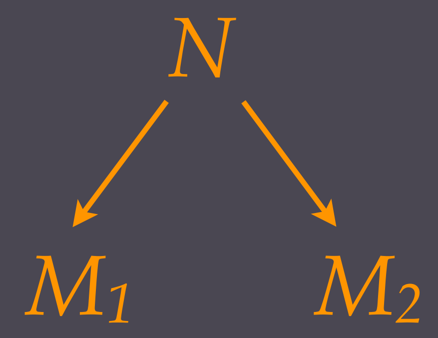
[Culler-Gordon-Luecke-Shalen '87]

- Cyclic Surgery Thm $\Longrightarrow$ at most 2 such covers
- [Gonzalez-Acuña—Whitten]+[Berge conjecture] $\Longrightarrow$ Berge knots are only examples

Q”: Can a hyperbolic knot complement be commensurable with another knot complement?

Yes! (see previous Q)

M_1 and M_2 are commensurable if there is a mfld N that is a finite cover of both:



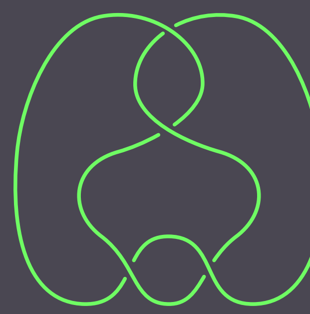
How many?

Conj. 1 [Reid—Walsh ’08]: There are at most 3 knot complements in a hyperbolic commens. class.

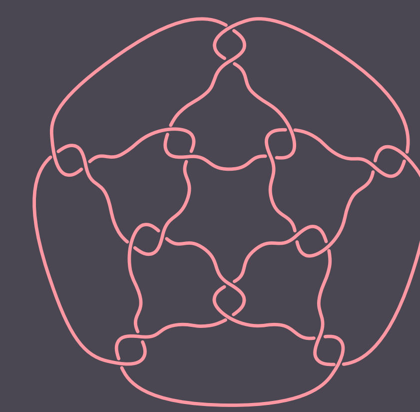
- *This bound is realized [Fintushel—Stern ’80, Hoffman ’10]*
- *[Boileau—Boyer—Cebanu—Walsh ’12]: Conjecture 1 holds for knots that don’t admit hidden symmetries*

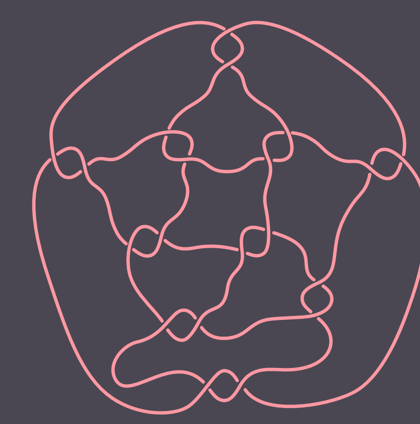
Def-n: M has a hidden symmetry if there is a symmetry of a finite index cover of M that is not a lift of a symmetry of M. $\Longleftrightarrow$ M irregularly covers a hyperbolic orbifold

Hidden symmetries appear to be rare among knots:



Conj. 2 [Neumann—Reid ‘92]: With the exception of the figure-8 knot and the two dodecahedral knots of Aitchison—Rubinstein, no hyperbolic knot complement has hidden symmetries.





- *Since Conj 1 is known to hold for the figure-8 knot and the two dodecahedral knots:*

Conj. 2 + [Boileau—Boyer—Cebanu—Walsh] $\Longrightarrow$ Conj. 1

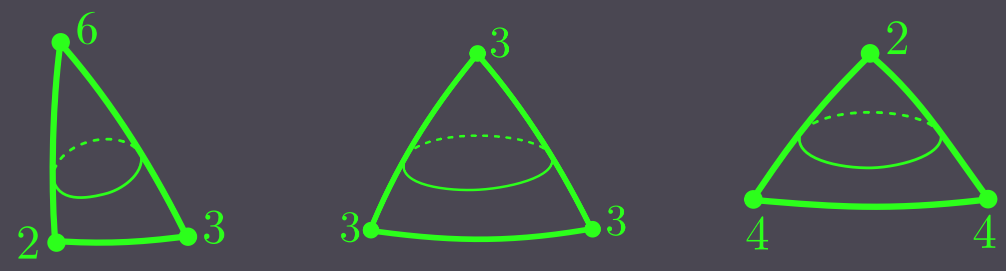
$\therefore$ try to prove Conj. 2

Neumann–Reid give an important criterion for knots:

A knot complement $M = S^3 \setminus K$ has hidden symmetries

$\iff$ M (irregularly) covers a rigid cusped orbifold.

orbifold with a cusp (cross-section) homeomorphic to one of these:

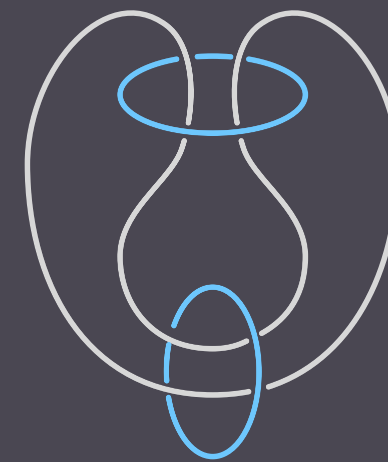


⇝ Goal:

try to find obstructions to knot complements covering rigid cusped orbifolds

Knots via Dehn filling:

Let L be the link complement shown:



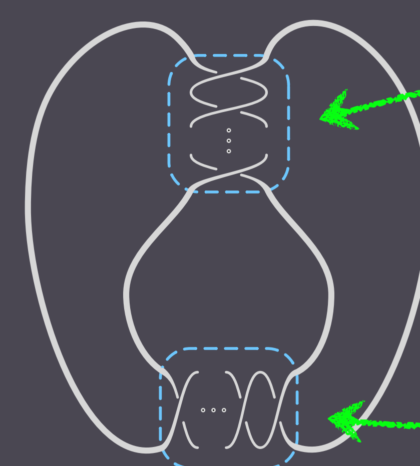
(Borromean rings)

(covers a rigid -cusped orbifold)

$(1,1)$-Dehn filling along the blue circles gives the figure-8 knot complement:

More generally...

$(1,n)$-Dehn filling gives $S^3 \setminus K_n =$



2n crossings each

- *Thurston: as* $n \longrightarrow \infty$, $S^3 \setminus K_n$ *converges geometrically to* $S^3 \setminus L$

Knots via Dehn filling:

Definition: Choose $\varepsilon > 0$ to be smaller than the shortest geodesic in N. Let M be a Dehn filling of some subset of the cusps of N. Then M is an *(ε, d_N)-twisted filling* of N if:

1. every core curve γ_i in M has length $< \varepsilon / d_N$
2. every other geodesic in M has length $> \varepsilon$

Covering Lemma:

(ε, d_N)-twisted filling $\Longrightarrow$ this cover exists

$$S^3 \setminus K \longrightarrow M$$

$$N \hookrightarrow M, \quad M \xrightarrow{p} Q, \quad N \dashrightarrow Q_0$$

$$Q \setminus \{\cup\, p(\gamma_i)\} = Q_0 \hookrightarrow Q$$

$Q_0 \cup$ rigid cusp $\Longleftarrow$ $Q \cup$ rigid cusp

Theorem 1 (*Hoffman—Millichap—W*): If $M = S^3 \setminus K$ is a hyperbolic knot complement that is an (ε, d_N)-twisted filling of a fully augmented link, then M has no hidden symmetries.

$(1,n)$-Dehn filling along red circles

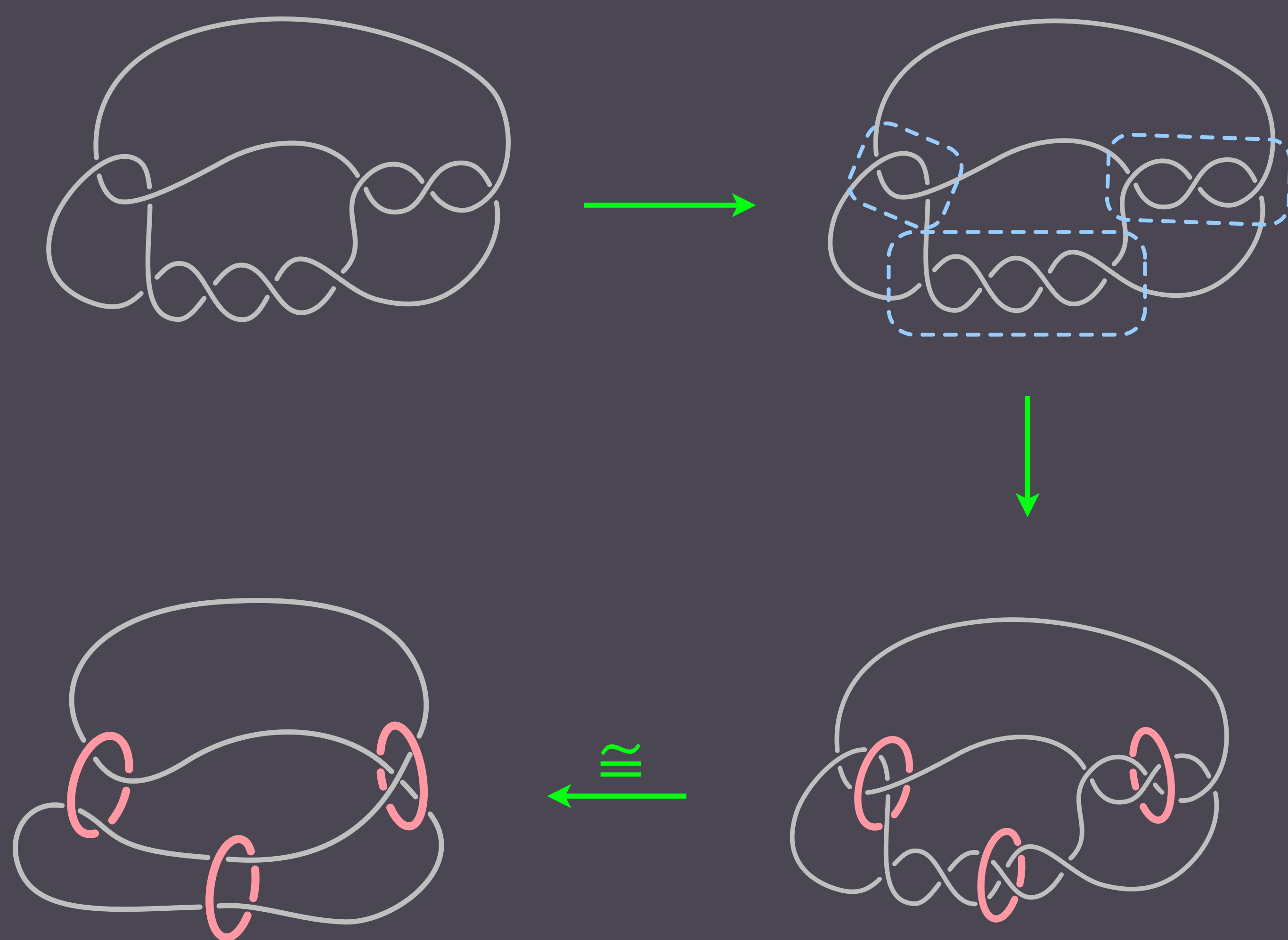
What's a fully augmented link (FAL)?

1) start with any knot:

2) partition crossings into twist regions

3) augment each twist region with unknot

4) reduce crossings by Rolfsen twists



FAL!

Covering Lemma $\Longrightarrow$

Theorem 2 *(CDHMMW)*: Let $\{S^3 \setminus K_i\}$ be a sequence of knot complements converging (geometrically) to $S^3 \setminus L$. If each knot $S^3 \setminus K_i$ covers a rigid cusped orbifold, then $S^3 \setminus L$ has hidden symmetries.

Covering Lemma + [Millichap-W '16] $\Longrightarrow$

Theorem 3 *(CDHMMW)*: If $N = S^3 \setminus L$ is a hyperbolic 2-bridge link, then at most finitely many orbifolds resulting from filling N are covered by knot complements with hidden symmetries.

Theorem 4 *(CDHMMW)*: The figure-8 knot complement is the only knot complement with hidden symmetries that covers a manifold with volume less than $6v_0$.

v_0 = volume of regular ideal tetrahedron ≈ 1

Thank you!