An algorithm for comparing Legendrian knots

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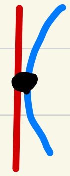
Joint work with M. Prasolov and V. Shastin

Legendrian = tangent to the standard contact structure

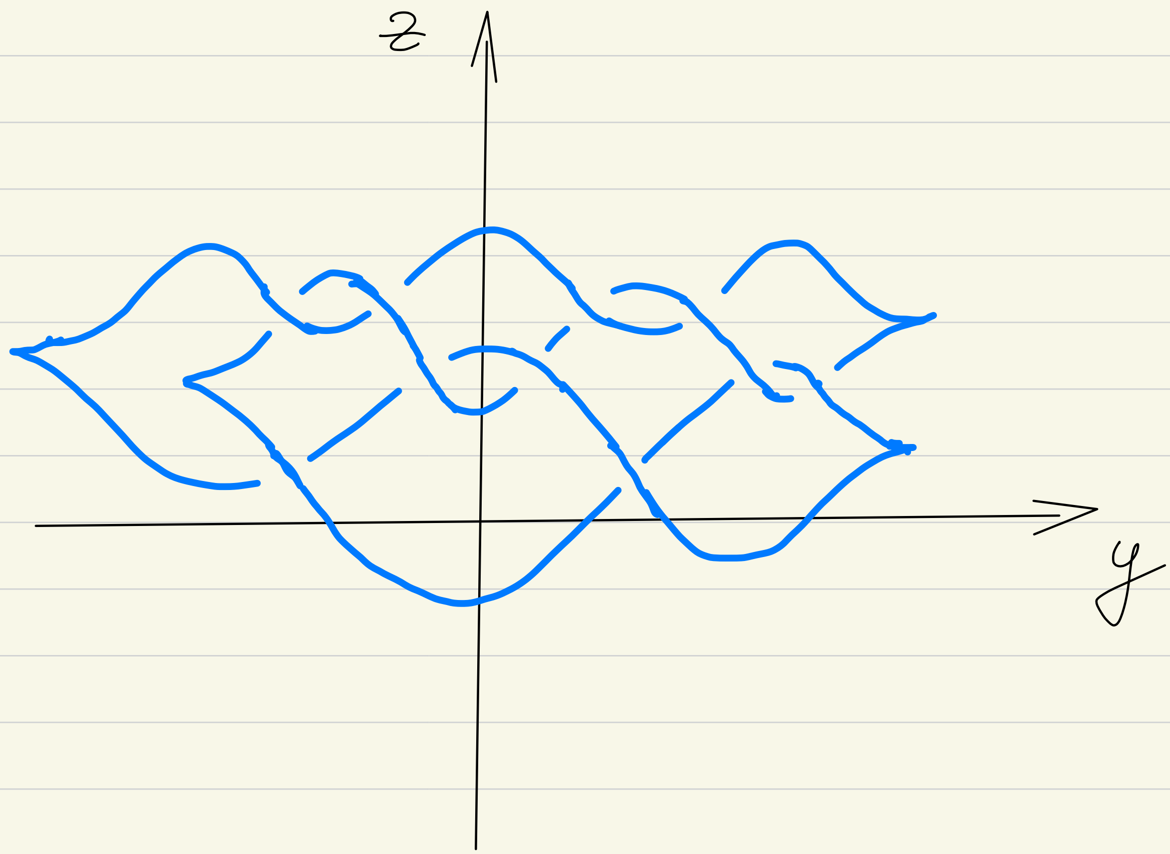
$$\xi_{st} = \ker(x\,dy + dz)$$

classification of Legendrian links $\approx$ classification of non-simplifiable rectangular (aka grid) diagrams of links

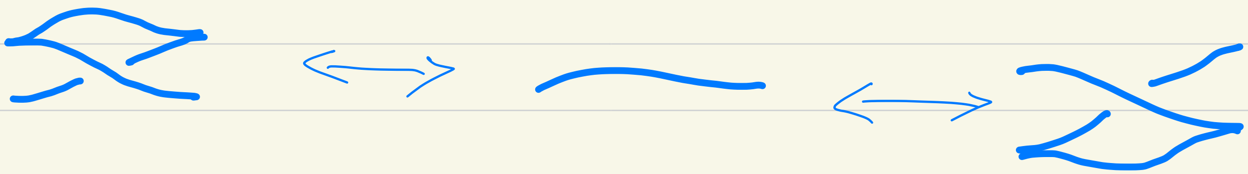
Front projections

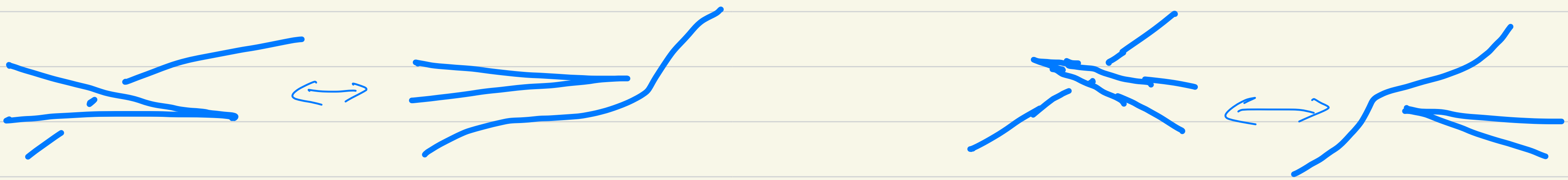
1) no 

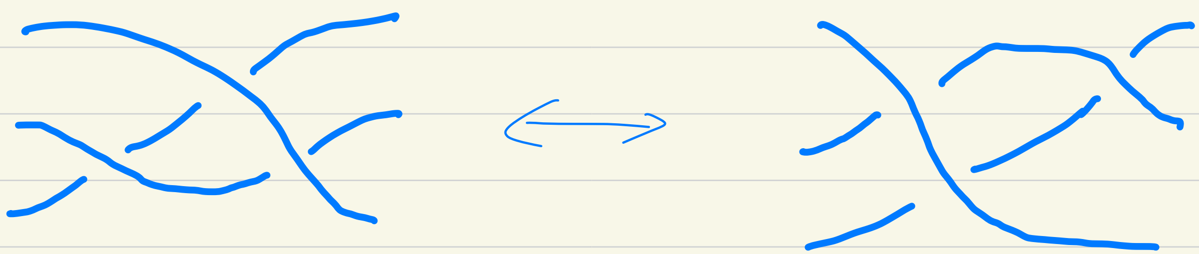
2) over/under acc. to slope:



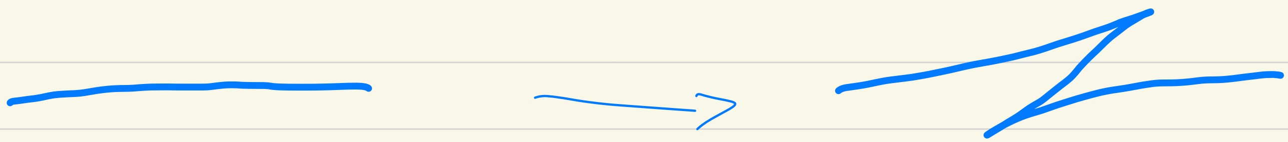
Reidemeister moves

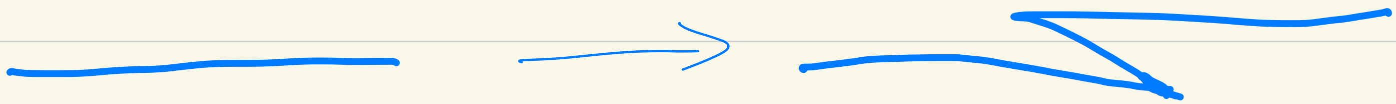




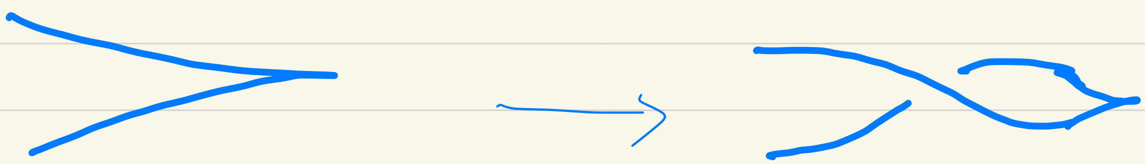


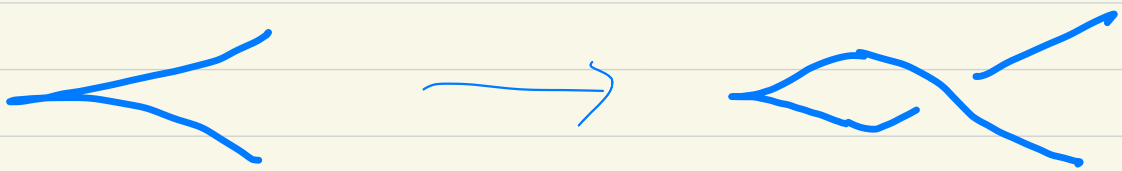
Stabilizations





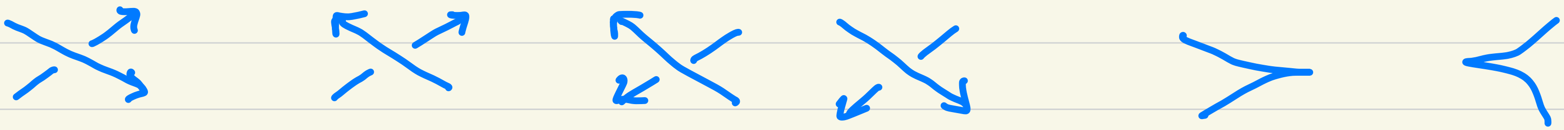
equivalently!





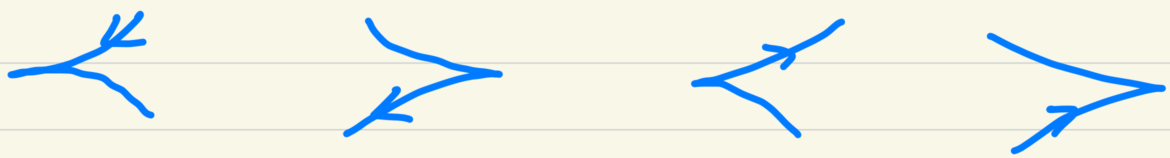
classical invariants

Thurston - Bennequin number tb



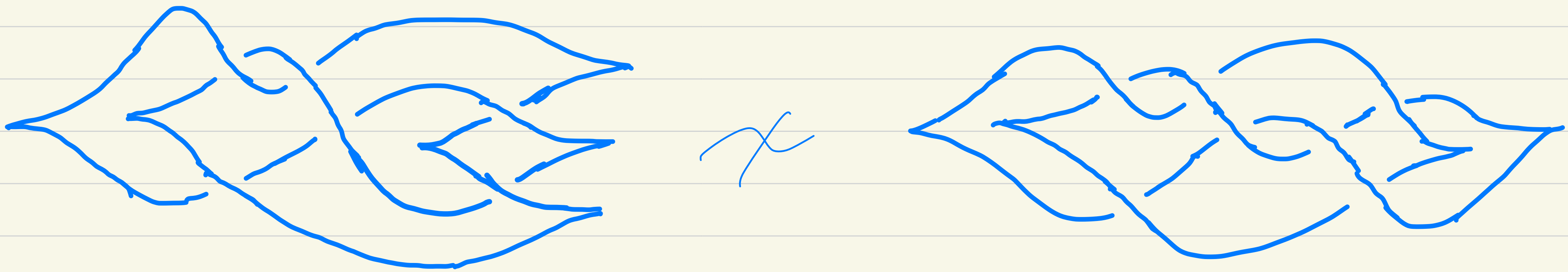
$+1$ $\quad -1$ $\quad +1$ $\quad -1$ $\quad -\frac{1}{2}$ $\quad -\frac{1}{2}$

Rotation (Maslov) number r



$+\frac{1}{2}$ $\quad +\frac{1}{2}$ $\quad -\frac{1}{2}$ $\quad -\frac{1}{2}$

Yu. Chekanov 1997:

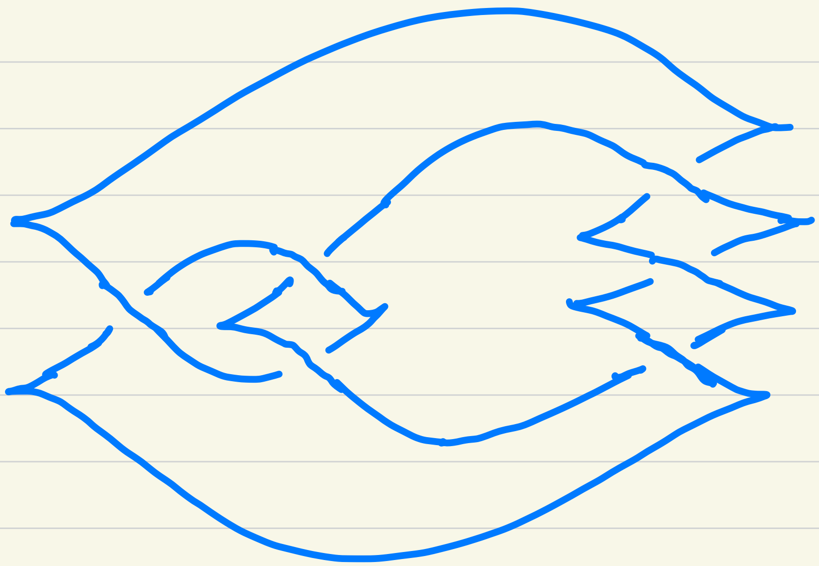


Ya. Eliashberg, D. Fuchs, L. Ng, P. Pushkar',
P. Ozsváth, Z. Szabó, D. Thurston, M. Fraser,
J. Etnyre, K. Honda, D. LaFountain, B. Tosun,
V. Vértesi

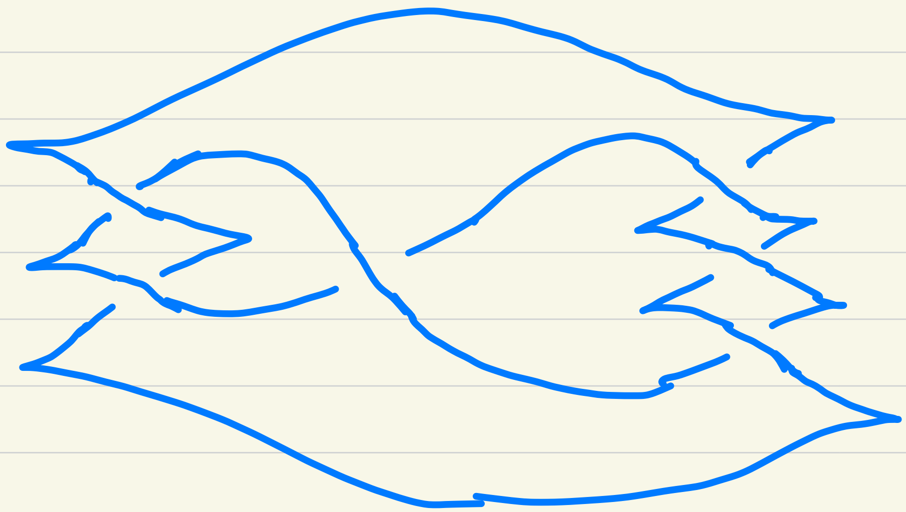
D. Bennequin, E. Giroux

W. Chongchitmate, L. Ng: an atlas

I.D., M.P. 2017:



$\not\cong$



I. D., M. P. 2016, 2017 ; I. D., V. Sh. 2018

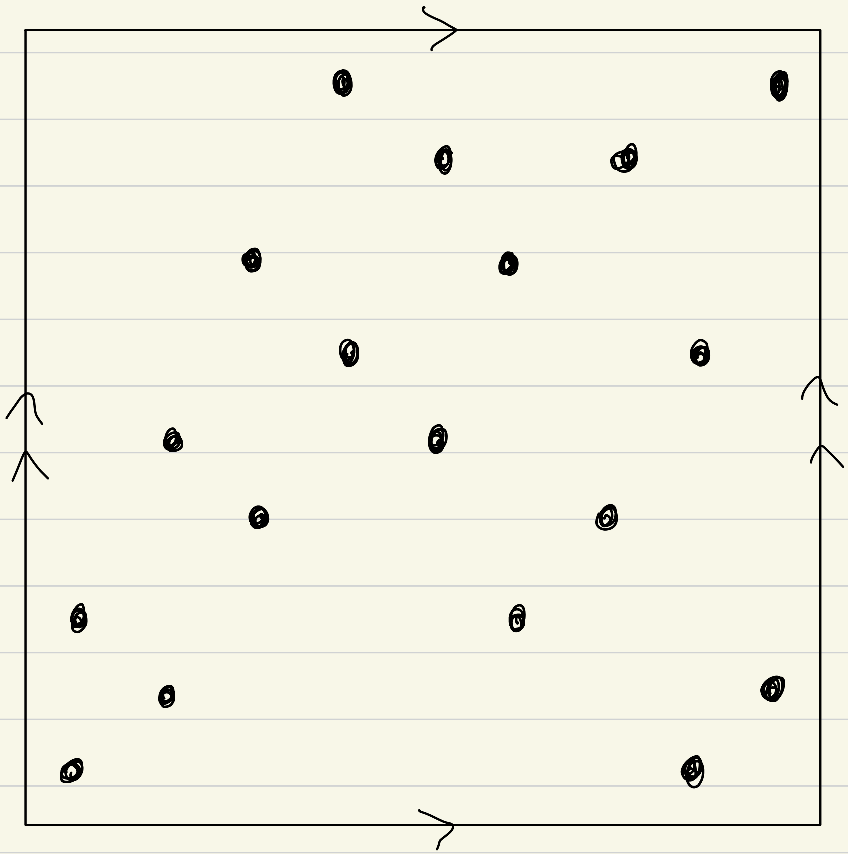
$\exists$ an annulus $A \subset S^3$ s.t.

A is tangent to ξ_{st} along ∂A, but

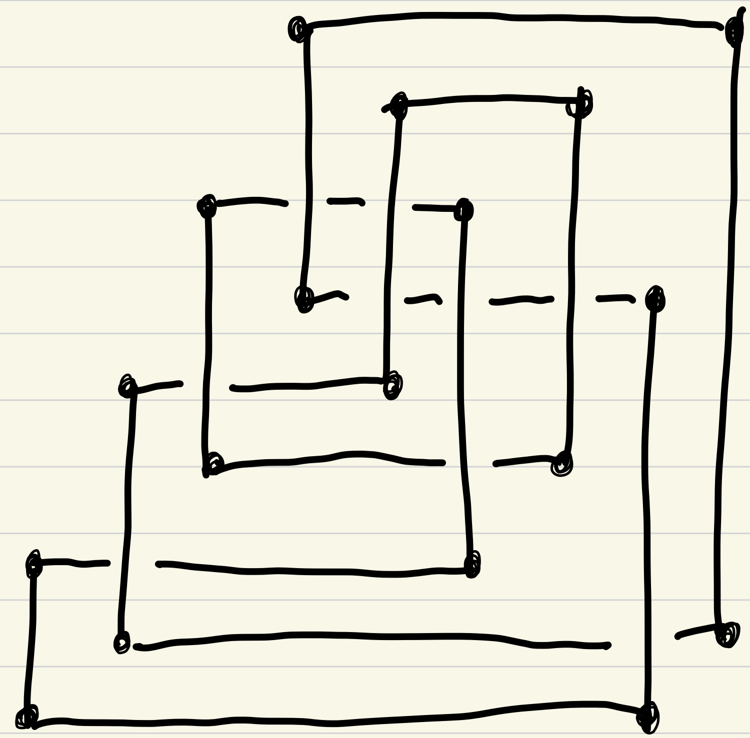
$\partial A = L_1 \sqcup L_2$, $L_1 \not\simeq L_2$ as Legendrian knots

disproving a claim in arXiv: 0909.4326

Rectangular diagram of a link

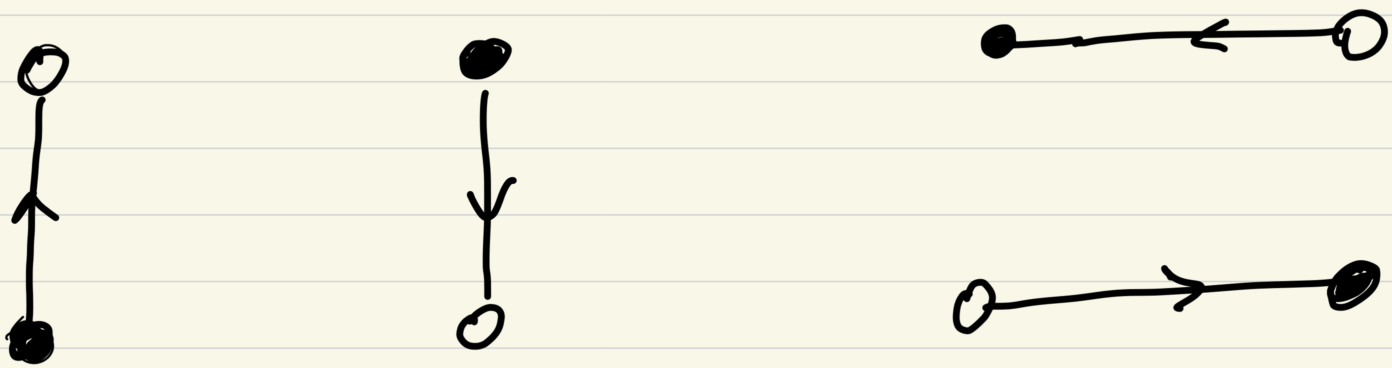


R



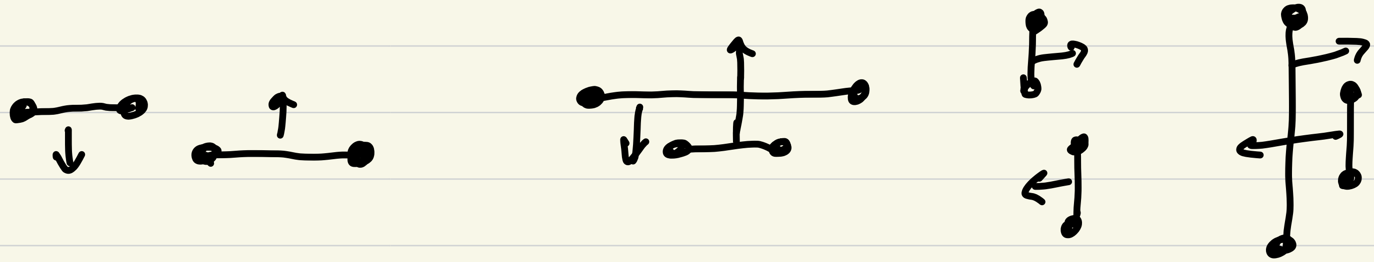
$\widehat{R}$

Orientation



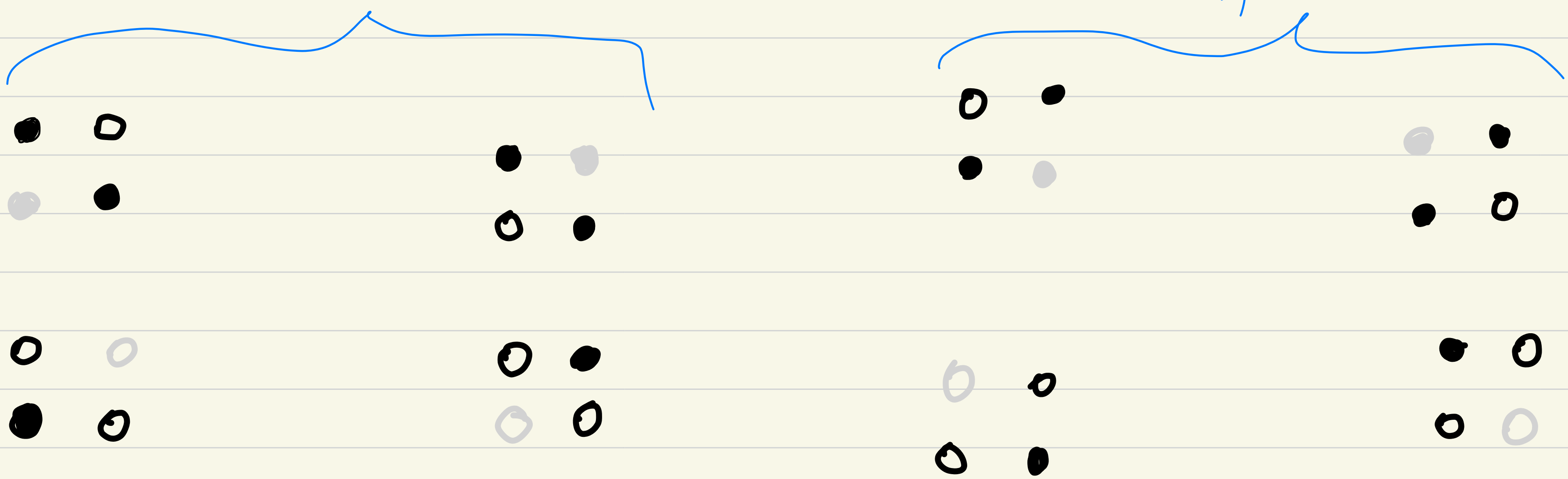
Moves

Exchange moves:



Stabilizations:

Type I Type II



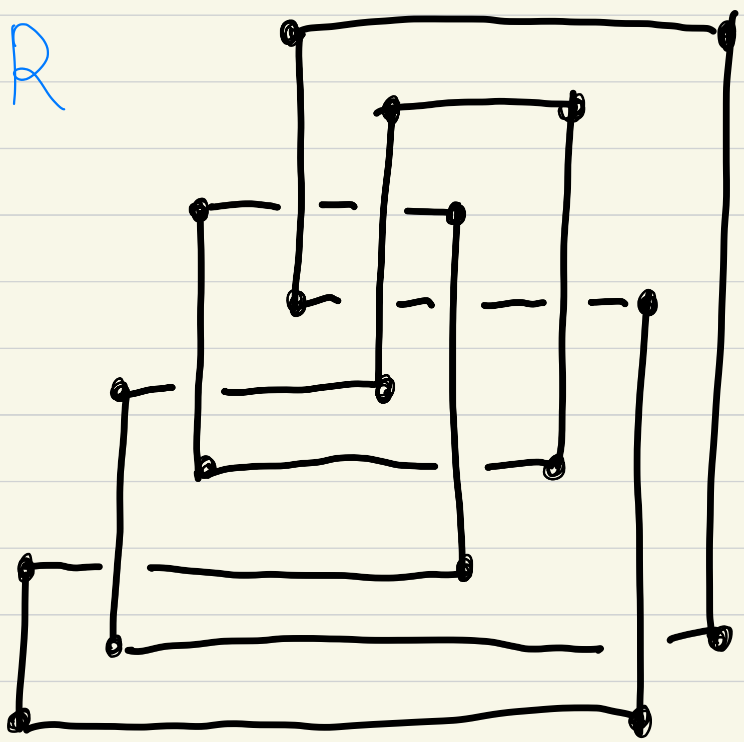
subtypes: $\overleftarrow{I}$ $\overrightarrow{I}$ $\overrightarrow{II}$ $\overleftarrow{II}$

For links, 'subtype of a stabilization' also includes a choice of a connected component

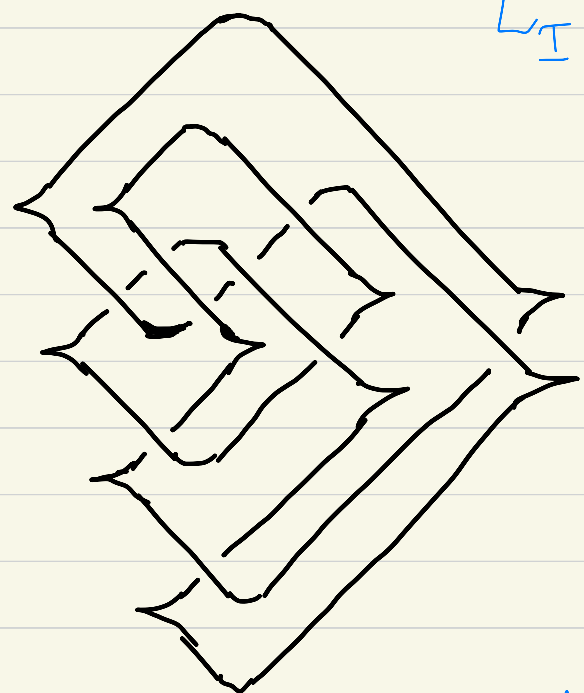
Stabilizations of the same subtype give the same result up to exchange moves

Rectangular diagram $\longrightarrow$ two Legendrian links

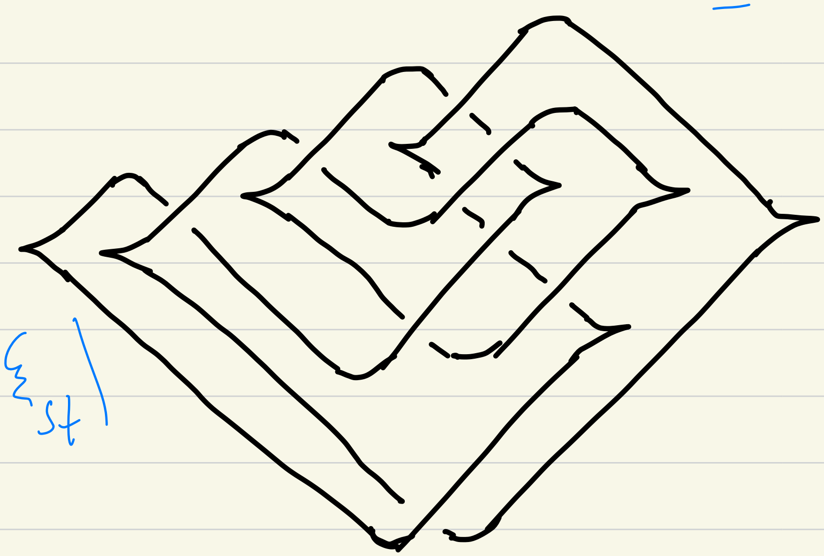
R



$L_{I}(R)$



$L_{II}(R)$



w.r.t. mirror image (ξ_{st})

Exchange moves preserve L_{I} and L_{II}.

Type I stabilizations preserve L_{I} and stabilize L_{II}

$\mathrm{I} \leftrightarrow \mathrm{II}$

Moves $\rightsquigarrow$ morphisms

$X_1 \mapsto X_2 \rightsquigarrow$ a homeo $(S^3, X_1) \to (S^3, X_2)$ preserving orientations, viewed up to isotopy.

Symmetry group $Sym(K) = \{\text{morphisms } K \to K\}$

Main results

① „Commutation" of type I and type II moves:

$$R_1 \overset{\text{any moves}}{\rightsquigarrow} R_2 \implies R_1 \overset{\text{only I}}{\rightsquigarrow} \overset{\text{only II}}{\rightsquigarrow} R_2,$$

same morphism

② $L_I(R_1) = L_{II}(R_2)$, $R_1 \overset{\cancel{\text{only exchanges}}}{\rightsquigarrow} R_2 \implies$

any seq. $R_1 \overset{\text{only I}}{\rightsquigarrow} R_2 \overset{\text{only II}}{\rightsquigarrow} R_1$ induces

a <u>nontrivial</u> elem. of $\mathrm{Sym}(\hat{R}_1)$

③ $\exists$ an algorithm for constructing a generating set of $\mathrm{Sym}(K)$

The algorithm

Q: $L_{\underline{T}}(R_1) = L_{\underline{T}}(R_2)$?

1. check whether R_1 and R_2 repr. the same topol. type

2. find $R_{\underline{1}} \overset{\text{only } \underline{T}}{\rightsquigarrow} R_{\underline{1}}' \overset{\text{only } \overline{T}}{\rightsquigarrow} R_2$

3. compute a gen. set for $\mathrm{Sym}(\hat{R_1'})$ $g_1, \ldots, g_m$

4. for each stab. subtype t of type $\underline{T}$, let
$$k(t) = \max_{i=1,\ldots,m} (\#\text{ of } t\text{-stab. in a realization of } g_i)$$
and apply $k(t)$ stabilizations

of subtype t to R_1' and R_2

$R_1' \rightsquigarrow R_1''$, $R_2 \rightsquigarrow R_2'$

(5) Check whether $R_1'' \xrightarrow[\text{only exchanges}]{} R_2'$

Tools: (a) rectangular calculus for surfaces in S^3

(b) Giroux convex surfaces