Braiding Groups of homeomorphisms of Cantor Sets

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In the last 5 years, there has been a confluence of researchers from Thompson groups and mapping class groups.

Goals:

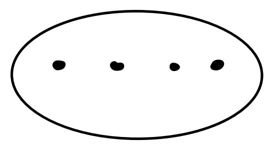
1) Explain connections between MCG's + Thompson Groups

2) Discuss why it makes sense to extend to other types of homeomorphism groups.

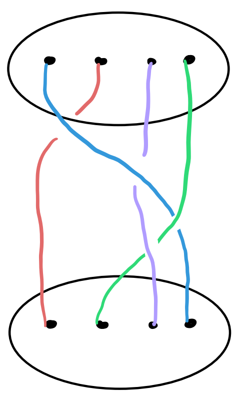
3) Finiteness properties

Def'n. For a surface S, the mapping class group of S is the set of orientation preserving homeomorphisms of S which fix the boundary pointwise, up to isotopy.

$$\mathrm{Map}(S)$$

Example: $S =$ 

$$\mathrm{Map}(S) = B_4 \quad \text{, braid group}$$

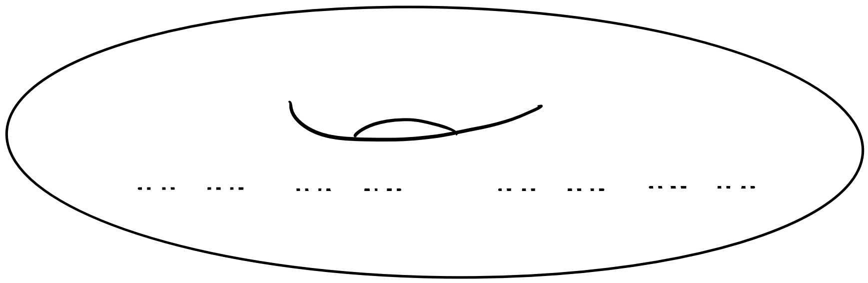


Tracking the movement of points with strands of a braid

Beginning with a blog post by Danny Calegari in 2014 there has been a shift to "big" mapping class groups

Map(S) where $\pi_1(S)$ not f.g.

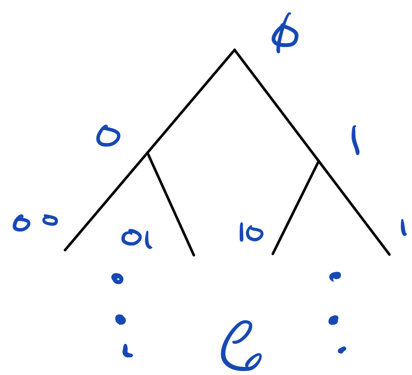
e.g. Take a surface, add a Cantor-space of punctures



Big mapping class groups less understood then "small" mapping class groups.

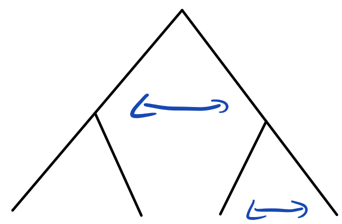
Way in: Well studied groups acting on Cantor sets.

Idea: Identify Cantor set with boundary of a tree



Self-similar groups:

$$\mathrm{Aut}\,T \cong (\cdots S_d \wr S_d) \wr S_d$$



Example 1: The Grigorchuk Group

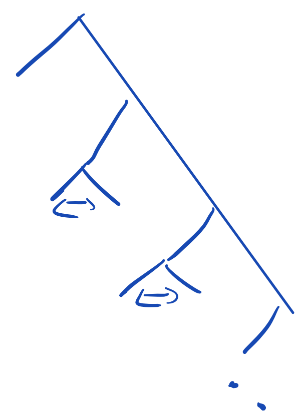
$a = (id, id)\,\sigma$

$b = (a, c)\,\_$

$c = (a, d)\,\_$

$d = (id, b)\,\_$

$b =$

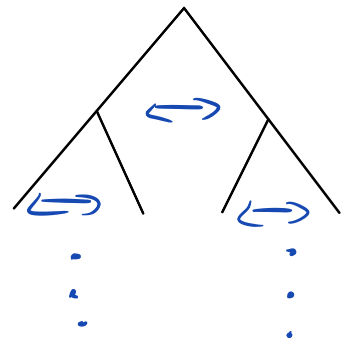


Groups where elements can be defined in terms of other group elements are self-similar.

Example 2: Self-identical groups

$\mathbb{Z}_2$

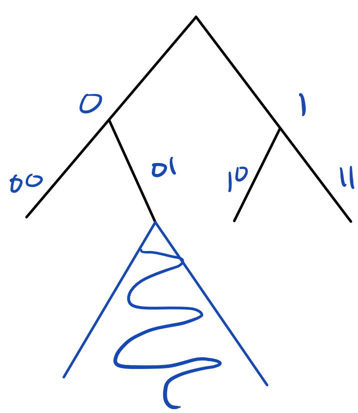
$g \to (g,g)\sigma$



Can do for any subgroup of S_d

<u>Higman</u> - <u>Thompson</u> <u>groups</u>

For a vertex v in the tree, define the cone at v to be the elements in $\mathcal{C}$ which pass through v



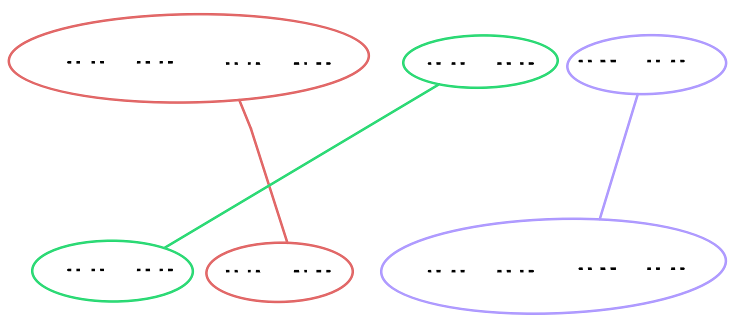
Infinite sequences with v as prefix.

$[v]$

Take $[u_1], \ldots, [u_t]$ and $[v_1], \ldots, [v_t]$ to be two partitions of $\mathcal{C}_0$ into cones.
Define $\varphi: X^\omega \to X^\omega$ via

$$u_i w \mapsto v_i w \quad \forall \; w \in X^\omega$$

The set of all such mappings is the Higman-Thompson group V_d

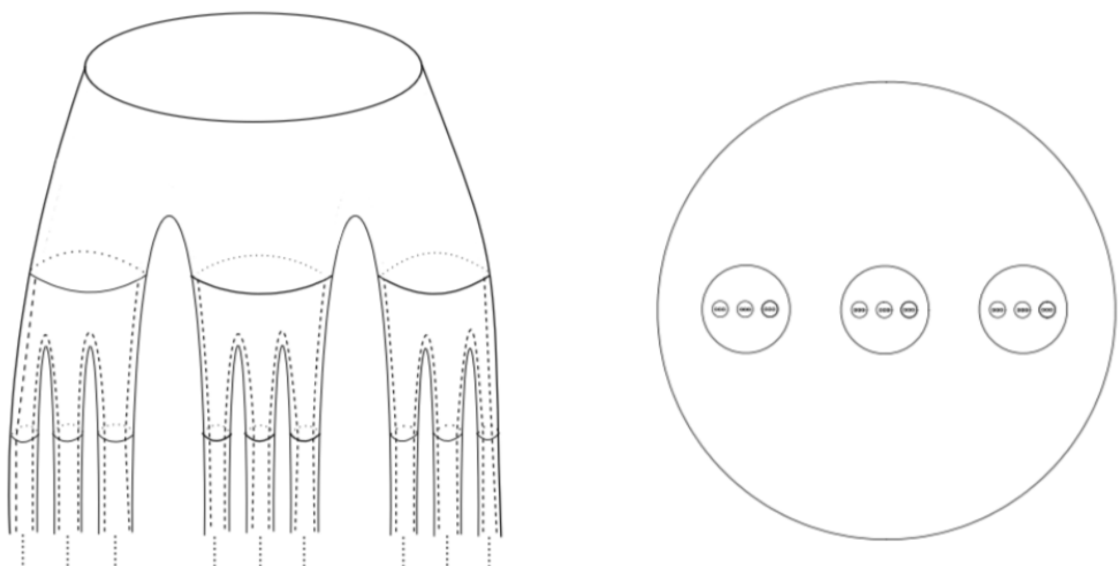


<u>Röver - Nekrashevych group</u>

$V_d(G)$: For G self-similar group the Röver-Nek gp is the group built from G & V_d

How to import these groups into mapping class groups?

replace permutations with braids.



Two models (with curves) for a disk punctured by the Cantor set

The braided versions of the above groups act by homeomorphisms which permute the curves.

A braided version of the Grigorchuk group - Allcock '20

Braided Higman-Thompson groups introduced by Brin ('07) Dehornoy ('06)

Braided Röver-Nekrashevych groups for self-identitical groups Aroca & Cumplido ('20)

Braided self-similar & general Röver-Nekrashevych groups S & Zaremsky ('21)

<u>Finiteness</u> <u>Properties</u>:

A group is of type F_n if it admits a classifying space with a compact n-skeleton

Type F_1 $\Longleftrightarrow$ finitely generated.

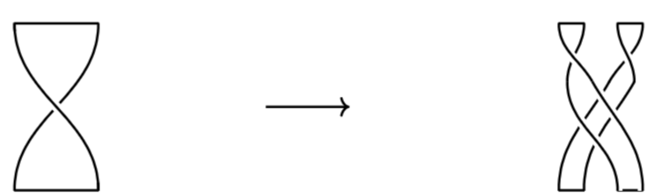
Type F_2 $\Longleftrightarrow$ finitely presented

Thm (S, Wu 2021) Let $H \le B_d$ and let G be the corresponding braided self identitical group. Then $Br V_d(G)$ is of type F_n if and only if H is.

Example: Dehn twist around boundary curve.

$$H \cong \mathbb{Z}$$

Ribbon - Higman Thompson group



Then Ribbon - Higman Thompson group is of type F_∞.

(S, Wu)
Note: Can be identified w/ asymptotic mapping class groups of Funar & Kapoudjian

Zaremsky: There exists a subgroup of B_d of type F_n but not F_{n+1} for each $0 \le n \le d-3$.

Thm (S, Zaremsky 21) The braided Grigorchuk group is not finitely presented. Nevertheless, the corresponding Braided Röver-Nek group has type F_∞.

Thanks