Math 151: Sections 1-3, 74.
Workshop 2: Limits and derivatives.

Problem 1. Sketch the graph of a function f with $f(0) = 1$, having a horizontal asymptote at $y = 2$, and having a horizontal asymptote at $y = 0$. Label the graph carefully. Is it possible for a function to have three distinct horizontal asymptotes? Explain why not, or give an example.

Problem 2. Sketch the graph of a function f where $\lim_{x\to 0^+} f(x) = +\infty$, and $\lim_{x\to 0^-} f(x) = 2$, and $\lim_{x\to +\infty} f(x) = 1$. Explain why your graph has the desired properties. Be sure to label the graph as necessary.

Problem 3. For each of the following limits, decide whether or not is exists. If the limit does not exist, explain carefully why not. If it does, find the limit and justify the correctness of your answer in your write-up of this problem. (If you use a calculator, check to see if the calculator gives a consistent answer for values of $|x - a|$ both large (10^{-3}) and small (10^{-15}).)

(1) $\lim_{x\to 1} \frac{3x^2+x+1}{x}$
(2) $\lim_{x\to 0^-} \frac{3x^2+x+1}{x}$
(3) $\lim_{x\to 1} \frac{x^2-1}{x-1}$
(4) $\lim_{x\to 0^+} \frac{x}{|x|}$
(5) $\lim_{x\to +\infty} \sin(x)$
(6) $\lim_{x\to 0} \frac{e^x-1}{x}$ (Calculator only.)

Problem 4. Let $f(x) = x^2$, the squaring function. Recall that the point/slope formula of the line with slope m and going through the point (x_0, y_0) is $y = y_0 + m(x - x_0)$.

(1) Give the point/slope formula for the secant line S_{ab} through the two points (a, a^2) and (b, b^2). Explain how you found this formula.
(2) Give the point/slope formula for the tangent line T_a to $f(x)$ at the point (a, a^2). Explain in detail how you found this formula using part (1) above. Do **not** simply say "the derivative of x^2 is ...".
(3) Recall that perpendicular lines have negative reciprocal slopes. Find the tangent line (to f) which is perpendicular to T_a, the tangent line at the point (a, a^2). What nice behaviour does $T_{1/2}$ have?

(Hint: First do everything for $a = 1$. Then, once you have a feel for the problem, do the general case.)