Math 151: Sections 1-3, 74.
Workshop 3: Continuity.

Problem 1. For each of the following statements either prove the statement is true (using the limit laws) or provide a counterexample:

(1) $\lim_{x\to 0} f(x) = f(0)$
(2) If $\lim_{x\to 0} f(x) = 3$ and $\lim_{x\to 0} g(x) = 3$ then $\lim_{x\to 0}(f(x) - g(x)) = 0$.
(3) If $\lim_{x\to 0} f(x) = \infty$ and $\lim_{x\to 0} g(x) = \infty$ then $\lim_{x\to 0}(f(x) - g(x)) = 0$.
(4) If $\lim_{x\to 0} f(x) = 0$ and $\lim_{x\to 0} g(x) = 0$ then $\lim_{x\to 0}(f(x)/g(x))$ is undefined.
(5) If $\lim_{x\to 0} f(x) = 3$ and $\lim_{x\to 0} g(x) = 3$ then $\lim_{x\to 0}(f(x)^2 - g(x)^2)/(f(x) - g(x)) = 6$.
(6) If $f(x) < g(x)$ for all x then $\lim_{x\to 0} f(x) < \lim_{x\to 0} g(x)$.

Problem 2. Let

$$g(x) = \begin{cases} 3x + 5, & x < -1 \\ ax + b, & -1 \le x \le 1 \\ 3x - 3, & x > 1. \end{cases}$$

Find values for a and b so that g is continuous. Explain precisely why your values for a and b make g continuous. Are there any other values of a and b making g continuous? Explain why or why not.

Problem 3. Graph $f(x) = x^2 - \cos(10\pi x) - 1$ in the standard window on your calculator (using radians). Note that the standard window shows exactly two roots, at plus and minus one.

(1) Prove, using the intermediate value theorem (IVT), that this function has a root between $x = 0$ and $x = 0.5$. (If you don't know what the IVT is, look it up in the textbook.) You do not need to use your calculator to do this.
(2) Explain, in detail, why f has no roots greater than $3/2$ in absolute value. (As we have seen, it does not suffice to simply graph the function.)
(3) How many roots does this function have? (Use your calculator.)
(4) Find, to four decimal places, the smallest positive root of $f(x)$.

Problem 4. A guru, as she has for many years, at the dawn of the fall equinox starts to walk the path up the mountain at the base of which her temple lies. By sundown she reaches the peak and spends the night contemplating the continuous nature of reality. At sunrise the next day she travels down the path and arrives at dusk to speak wisdom:

> No matter how I
> Climb, twice there is a single
> Place, at the same time.

Can you help her disciples unravel the guru's words?