

**Math 151: Sections 1-3, 74.**

**Workshop 4: Derivatives.**

**Problem 1.** Let  $f(x) = x^2$ . For every  $a$  let  $N_a$  be the *normal line* – the line through  $(a, a^2)$  which is perpendicular to the tangent line  $T_a$ . (Here we are using the same notation as in workshop number 2, problem 4). Give the point/slope formula for  $N_a$ . Explain how you found this formula. Sketch a graph showing  $f$ ,  $T_1$ , and  $N_1$ .

**Problem 2.** Write a short story about the parabola  $y = x^2$ . Use  $S_{ab}, T_a, N_a$  as supporting characters. Ok, just kidding. Go on to the next question which doesn't involve  $x^2$  at all. I promise.

**Problem 3.** Compute the following derivatives directly from the definition:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}.$$

For every step, give a short explanation.

- (1)  $f(x) = x^2 + 5$
- (2)  $f(x) = x/|x|$
- (3)  $f(x) = x/(x^2 + 1)$
- (4)  $f(x) = \sqrt{x^2 + 9}$

**Problem 4.** Let

$$g(x) = \begin{cases} -x, & x < 0 \\ ax^2 + bx + c, & x \geq 0. \end{cases}$$

Find all values for  $a, b, c$  so that the functions  $g$  and  $g'$  are both continuous. Explain your work. Provide at least three possible graphs of  $g$ , each showing different behaviour.

**Problem 5.** Consider the two curves  $S : y = x^2$  and  $P_C : y = C - x^2$  (here  $C$  is a constant to be determined later).

- (1) Sketch the curves  $S$  and  $P_C$  for  $C = -1, 0, 1, 2$  in a single coordinate plane. Discuss how the number of intersections of  $S$  and  $P_C$  varies with the choice of the constant  $C$ .
- (2) Find expressions for the points of intersection of  $S$  and  $P_C$ , in terms of the constant  $C$ . Carefully explain your work and draw an explanatory picture.
- (3) Find a value of  $C$  so the curves  $S$  and  $P_C$  have two intersection points so that, at each intersection point, the lines tangent to the two curves are perpendicular. Provide a sketch of the tangent lines and both curves for this value of  $C$ . Explain how you found  $C$ . (Such pairs of curves arise in the study of electromagnetism.)