Math 151: Sections 1-3, 74.
Workshop 5: More Derivatives.

Problem 1. Let

$$g(x) = \begin{cases} 3x + 5, & x < -1 \\ ax^3 + bx^2 + cx + d, & -1 \leq x \leq 1 \\ 3x - 3, & x > 1. \end{cases}$$

Find values for a, b, c, d so that the functions g and g' are continuous. Are there any other values of a, b, c, d making g and g' both continuous? Explain and provide a graph of g. (This method of connecting curves is sometimes called taking a *cubic spline*.)

Problem 2. An argument c is called a *root* for a function f if $f(c) = 0$. Graphically, a root is a place where the graph of f meets the x-axis.

(1) Find the root of the linear functiony $y = mx + b$.
(2) The *degree* of a polynomial is the largest exponent appearing. For example, linear functions have degree one while $x^2 + x + 1$ has degree two. Find polynomials of degree zero, two, four, and six which have no roots. Explain why your examples work.
(3) Discuss the number of roots of the function $x^3 + x + 1$. Give each root to at least two decimal places and describe your method. Check your work by employing another, different method.
(4) Does every polynomial of odd degree have a root? If you think so, give a convincing explanation why. If you think not, explain your counterexample.

Problem 3. Given that the derivative of $f(x)$ is $f'(x)$ compute the derivatives of:

(1) $x^2 f(x)$
(2) $\frac{f(x)}{x^2}$
(3) $\frac{x^2}{f(x)}$
(4) $\frac{f(x)-1}{f(x)+1}$

Problem 4. Let $f(x) = x^3 - 3x + 2$. Graph $f(x)$.

(1) Give the point/slope formula for every tangent line to the graph of f that is parallel to the line $y = 3x + 1$. Explain why you have found all such tangent lines.
(2) Is there a tangent to f which is parallel to no other tangent to f? If so, find all such. If not, explain why.