Math 151: Sections 1-3, 74.
Workshop 7: Implicit differentiation and ellipses.

Problem 1. On composing with the function $1/x$:

(1) Sketch the graphs of the functions $x+1$, $\frac{1}{x}+1$, $\cos(x)$, and $\cos(1/x)$. Note any horizontal or vertical asyptotes of these graphs.
(2) Sketch a graph of $F(x) = e^x$, find its derivative, give an equation for the line tangent to the curve at $x = 2$, and sketch on the same graph that tangent line. Note any horizontal or vertical asymptotes on the graph.
(3) Sketch a graph of $G(x) = e^{\frac{1}{x}}$, find its derivative, give an equation for the line tangent to the curve at $x = 2$, and sketch on the same graph that tangent line. Note any horizontal or vertical asymptotes on the graph.
(4) Suppose that $h(x)$ is continuous and defined for all real numbers x. Suppose also that $\lim_{x\to\infty} h(x) = \infty$ and $\lim_{x\to-\infty} h(x) = \infty$. Sketch and explain a possible graph for $h(1/x)$. Note any horizontal or vertical asymptotes on the graph.

Problem 2. Discuss with your group how to graph the collection of points in the xy plane which satisfy $E : x^2 - xy + y^2 = 3$. Here are a few ideas to think about: solving for y, making a table of values, or messing about with your calculator.

Now find a *bounding box* for the figure E with sides parallel to the axes: that is, find the rectangle with vertical sides and horizontal top and bottom so that, first, E is contained in the rectangle and, second, there is no smaller rectangle containing E. To visualize this copy your graph of E and sketch where the box should be. Now find equations for all four sides of the box, the (x, y) coordinates of the corners of the box, and the lengths of the sides of the box.

(As a challenge, can you find a smaller bounding box? What do you think the smallest bounding box is? Can you prove your guess is correct?)

Problem 3. As an addendum to problem number one, graph the curve W : $1/x^2 - y/x + y^2 = 3$. Explain how you found the graph. Note any horizontal or vertical asymptotes on the graph. Label the places where the graph crosses the x axis.

Problem 4. Suppose that a point moves about in the plane, with x coordinate at time t being $\cos(t)$ and y coordinate at time t being $\frac{1}{2}\sin(t)$. Draw the path of the point. It may help to construct a table with three columns: one for t, one for x, one for y.

(1) When is the point closest to the origin $(0, 0)$? When is it farthest?
(2) How fast is the point moving in the x direction at these places? How fast is the point moving in the y direction at these places? Record this data in a table with three columns: one for t, one for dx/dt, and one for dy/dt.
(3) Note that, at every time recorded in your table, the point is always moving at zero velocity in either the x or the y direction. (If this is not the case you have made a mistake.) Either explain why this always happens for all t or find a time t where the point has non-zero velocity in both the x and y directions.