

Math 151: Sections 1-3, 74.

Workshop 9: Optimization and others.

Problem 1. Let P be the curve $y = 1 - x^2$ for values of x between -1 and $+1$. Find the largest rectangle R which has one edge on the x -axis and has two vertices on the curve P . Explain your work. Perhaps you could give several labeled pictures, showing various possibilities for the rectangle R .

Problem 2. Sketch the graph of $y = \sqrt{\frac{x}{x-5}}$. Label all roots, extrema, inflection points, asymptotes, *etc.* Also comment on any symmetries of the graph, if any. It may help to first sketch the graph of $y = \frac{x}{x-5}$.

Problem 3. Let P be the parabola $y = 1 - x^2$. What is the point (a, b) (with $a \geq 0$) on P which is closest to the origin $(0, 0)$? As always explain your work and draw a picture. (Here is a hint which may make the problem easier: a positive differentiable function f is minimized exactly when f^2 is minimized. Can you see why this is?)

Problem 4. (Shamelessly stolen from Kevin Hartshorn, of Moravian College.) Consider the two functions $f(x) = 1 - x$ and $g(x) = 1/x$. We can compose these two functions with each other in two ways:

$$f(g(x)) \text{ and } g(f(x)).$$

We can go further and compose these two new functions with themselves, and also with the original ones, in a number of ways. We can also compose functions with themselves, like $f(f(x))$. You might think that you'll just keep generating more and more new functions.

Surprisingly, starting with this f and g , only a finite number of new ones get generated by composition, even though there may be many different ways of composing f and g to get the same function. Remember that two very different looking formulae may represent the same function.

- (1) How many distinct functions can you find, including f and g themselves? List them.
- (2) How do you know that these are all there are? Can you express them all as some composition of the functions f and g ? In more than one way?
- (3) Is there some way of arranging or displaying the functions you generate that helps to answer the previous two questions?

(As a tough at-home-challenge you could try answering this same three questions but starting with the pair of functions $f(x) = 2 - x$ and $g(x) = 1/(1 - x)$. How do the answers change?)