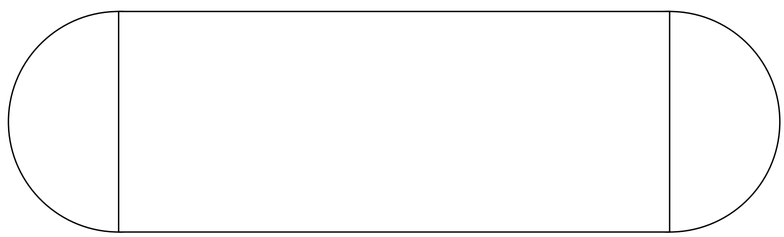
Math 151: Sections 1-3, 74.
Workshop 10: The exam and antiderviatives.

Problem 1. Find two positive numbers whose product is 100 and whose sum is a minimum. Explain all steps.

Problem 2. We are asked to construct a race track of total length exactly 400 yards. The track is to enclose a field which has the following shape: A rectangle with one semicircle attached to each end. (See the figure below.) What is the shape of the track which maximizes the area of the rectangle?



Problem 3. Here is the classic "can" problem:

(1) Twelve US fluid ounces is about 355 cubic centimeters (or so says the Google calculator). Find the dimensions in centimeters of a cylinder (of height h and radius r) which contains 12 fluid ounces while minimizing surface area. What is the ratio h/r?
(2) Suppose that the volume is V cubic centimeters. Find h and r (in terms of V) for the can which contains V cubic centimeters while minimizing the surface area. What is the ratio h/r?
(3) Go home (at the end of the period) and find at least three canned goods. Jolt, for example. Measure h and r in centimeters for each of these and record h, r, and the ratio h/r. (Explain how you found r – it is not so easy to find the center of a circle!) How does this compare to the ratios h/r you found above? Are they bigger or smaller? How do you explain the difference?

Problem 4. Let $f(x) = x/e^x$. Sketch the graph of f showing all roots, extrema, inflection points, asymptotes, *etc.* In addition, record the first few derivatives of f. Use the patterns you notice to write down a formula for the nth derivative of $f(x)$, $f^{(n)}(x)$. Find the graph of $f^{(n)}(x)$. Use the pattern of the derivatives to guess antiderivatives (first, second, third, *etc*) for the function f. Finally, check that your guess is correct by differentiating.