Math 151: Sections 1-3, 74.
Workshop 11: Fun, fun, fun!

Problem 1. Let $f(x) = \sqrt{1-x^2}$, defined on the interval $[0,1]$.

(1) Give an accurate sketch of the graph of f.
(2) Break the interval $[0,1]$ into four subintervals and compute $\sum_{i=1}^{4} f(x_i^*) \cdot \frac{1}{4}$ where x_i^* is the right endpoint of the ith subinterval. Give a sketch of the four rectangles giving the Riemann sum.
(3) Break the interval $[0,1]$ into four subintervals and compute $\sum_{i=1}^{4} f(x_i^*) \cdot \frac{1}{4}$ where x_i^* is the left endpoint of the ith subinterval. Give a sketch of the four rectangles giving the Riemann sum.
(4) Break the interval $[0,1]$ into 100 subintervals and compute $\sum_{i=1}^{100} f(x_i^*) \cdot \frac{1}{100}$ where x_i^* is the right endpoint of the ith subinterval. (Hint: Program your calculator to do this sum for you. If you can't, skip this step.)
(5) Break the interval $[0,1]$ into 100 subintervals and compute $\sum_{i=1}^{100} f(x_i^*) \cdot \frac{1}{100}$ where x_i^* is the left endpoint of the ith subinterval. (Hint: Program your calculator to do this sum for you. If you can't, skip this step.)
(6) Rcalling that the area of the unit circle is π use your answers to the above questions to find upper and lower bounds for π.

Problem 2. Suppose that $f(x) = \sqrt{1/x}$, for positive values of x.

(1) Find the point (a,b) on the graph of f which is closest to the origin; that is, which minimizes the distance between the graph and the point $(0,0)$. (Hint: to minimize a distance, it is enough to minimize the square of the distance.)
(2) On your graph of f draw the straight line connecting $(0,0)$ to (a,b). Now compute the slope of this line (b/a) and also its negative reciprocal $-a/b$.
(3) Find $f'(a)$.
(4) What do you notice? Explain what this means geometrically.

Problem 3. Let P be the parabola with function $f(x) = 1 - x^2$. Graph P. Find the points of P which are closest (farthest) to (from) the origin. As above, it is enough to minimize (maximize) the square of the distance. (You may find it makes the problem much easier to use implicit differentiation.)

For each of these points compute the ratio $-x/y$ and compare to the slope of the tangent line to E at the point. Does the same thing (as in the previous problem) happen? Will this always happen? Can you explain why using either calculus or geometry?