Math 151: Sections 1-3, 74.
Workshop 12: Integrals and the FTC.

Problem 1. Graph the function $f(x) = 1/(x^2+1)$. Be sure to note any extrema, inflection points, asymptotes, etc.

(1) What is the largest value of $f(x)$ on the interval $[0, 2]$? The smallest?
(2) Use your answers to the above and the geometric meaning of the definite integral (as an area) to show that

$$0.4 \leq \int_0^2 \frac{1}{x^2+1} dx \leq 2.$$

(3) By cutting the interval $[0, 2]$ into two pieces and repeating the above two steps show that

$$0.7 \leq \int_0^2 \frac{1}{x^2+1} dx \leq 1.5.$$

(4) By cutting the interval $[0, 2]$ into four pieces and repeating the above two steps find even better lower and upper bounds on $\int_0^2 \frac{1}{x^2+1} dx$. Give a picture showing the graphical intepretation.
(5) Using your calculator's "fnInt(" program, estimate the numerical value of the integral. (This program can be found in the catalog – press SECOND and then the zero key. Scroll down to "fnInt(" and hit ENTER. Now type $1/(X^2+1), X, 0, 2)$ and hit ENTER. This should compute the integral numerically. Note that, by the fundamental theorem of calculus, this definite integral should have the value arctan(2).

Problem 2. (1) A car is traveling at 50 mi/hr when the brakes are fully applied, producing a constant deceleration of 40 ft/sec^2. What is the distance covered before the car comes to a complete stop?
(2) A car braked with a constant deceleration of 40 ft/sec^2 and produced skid marks measuring 160 ft before coming to a stop. How fast was the car traveling when the brakes were first applied?

Problem 3. Let the function $f(x)$ be defined by

$$f(x) = \begin{cases} x, & 0 \leq x \leq 1, \\ 1, & 1 \leq x \leq 2, \\ 3-x, & 2 \leq x \leq 3. \end{cases}$$

(1) Draw the graph of $y = f(x)$.
(2) Let $0 \leq b \leq 3$. Define $F(b)$ to be the area under the graph of $y = f(x)$ between $x = 0$ and $x = b$. Use elementary geometry to find a piecewise defined formula for $F(b)$. Draw the graph of $F(b)$.
(3) Calculate $F'(x)$ for $0 \leq x \leq 3$. Draw the graph of $F'(x)$. What other graph does this look like? Why?

Problem 4. Let the function $g(x)$ be defined by

$$g(x) = \begin{cases} x, & 0 \leq x < 1, \\ 2, & x = 1, \\ x, & 1 < x \leq 2. \end{cases}$$

(1) Draw the graph of $y = g(x)$.

(2) Let $0 \leq b \leq 2$. Define $G(b)$ to be the area under the graph of $y = g(x)$ between $x = 0$ and $x = b$. Use elementary geometry to find a formula for $G(b)$. Draw the graph of $G(b)$.

(3) Calculate $G'(x)$ for $0 \leq x \leq 2$. Draw the graph of $G'(x)$. What other graph does this look like? Are these graphs indentical? Why not? Why does this *not* contradict the fundamental theorem of calculus?