Math 151: Sections 1-3, 74.
First Midterm.

Name:
Discussion section meeting time:

Print your name and discussion section time in the space provided. Please turn off all cell phones, pagers, music devices, etc. You may not use notes, books, or calculators. (You may use a ruler.) Read each problem carefully and note that Problem **7** spans two pages. When your final answer is a number or a formula, please draw a box around it. Correct answers without justification will receive no credit. Write neatly: we cannot grade illegible papers. Partial credit will be given for incomplete answers which display some understanding of the problem.

There are 8 problems. You have 80 minutes to work. Good luck!

Problem	1	2	3	4	5	6	7	8	Total
Poss.	10	10	15	10	10	10	25	10	100
Score									

Problem 1. (10 points) Find the exact value of each of the following expressions: $e^{2\ln 4}$ and $\tan(\arcsin(1/2))$.

Problem 2. (10 points) Sketch and label the graph of a function f that satisfies all of the given conditions: $\lim_{x\to 3^+} f(x) = 4$, $\lim_{x\to 3^-} f(x) = 2$, $\lim_{x\to -2} f(x) = 2$, $f(3) = 3$, $f(-2) = 1$.

Problem 3. (5 points each) Evaluate the following limits. Show all work. Justify all steps. (Answers using l'Hospital's rule will receive no credit.)

(1) $\lim_{x \to 3} \frac{\sqrt{x+1}-2}{x-3}$

(2) $\lim_{x \to \infty} \frac{3x^2 - x + 4}{2x^2 + 5x - 8}$

(3) $\lim_{x \to 0} \frac{\sin x}{5x}$

**Problem 4.** (10 points) Find all asymptotes of the function
$$f(x) = (4 + x^2)/(4 - x^2).$$
Use this information to sketch the graph of f. Be sure to label your sketch, including the places where the graph crosses the x and y axes.

Problem 5. (10 points) Set $f(x) = x + \sqrt{x}$. Compute $f'(x)$ directly from the definition of the derivative.

Problem 6. (10 points)
   The graph of the function f is given. Sketch the graph of f' below it.

PSfrag replacements
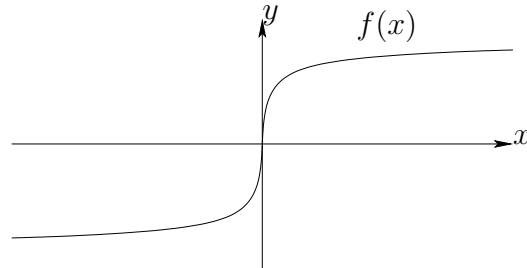
Problem 7. (5 points each) Differentiate the following functions however you like. Explain all steps.

(1) $f(x) = x^2 e^x$

(2) $f(x) = \frac{\cos x}{x^2}$

(3) $f(x) = (x^3 + 4x)^7$

(4) $f(x) = \ln(\sin x)$

(5) $f(x) = \sin(\ln x)$

Problem 8. (10 points) Let $P : y = x^2$ be the standard parabola in the xy plane. Let E_c be the family of ellipses $E_c : 2y^2 + x^2 = c$ (for $c > 0$). Sketch and label a graph showing P, E_1, and E_4. Also, show that every ellipse E_c is orthogonal to the parabola P. (Hint: you do *not* need to explicitly compute the intersection points of P and E_c to solve this problem.)