

Problem 1. (10 points) Find the exact value of each of the following expressions: $e^{2\ln 3}$ and $\tan(\arccos(1/2))$.

Problem 2. (10 points) Sketch and label the graph of a function f that satisfies all of the given conditions: $\lim_{x \rightarrow 3^+} f(x) = 4$, $\lim_{x \rightarrow 3^-} f(x) = 2$, $\lim_{x \rightarrow -2} f(x) = 2$, $f(3) = 3$, $f(-2) = 1$.

Problem 3. (5 points each) Evaluate the following limits. Show all work. Justify all steps. (Answers using l'Hospital's rule will receive no credit.)

(1) $\lim_{x \rightarrow 7} \frac{\sqrt{x+2}-3}{x-7}$

(2) $\lim_{x \rightarrow \infty} \frac{4x^2-x+4}{2x^2+5x-8}$

(3) $\lim_{x \rightarrow 0} \frac{\sin x}{3x}$

Problem 4. (10 points) Find all asymptotes of the function

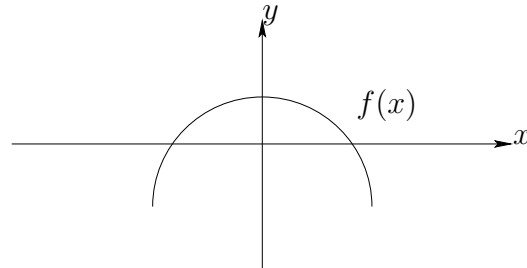
$$f(x) = (x^2 + 4)/(x^2 - 4).$$

Use this information to sketch the graph of f . Be sure to label your sketch, including the places where the graph crosses the x and y axes.

Problem 5. (10 points) Set $f(x) = 2x + \sqrt{x}$. Compute $f'(x)$ directly from the definition of the derivative.

Problem 6. (10 points)

The graph of the function f is given. Sketch the graph of f' below it.



Problem 7. (5 points each) Differentiate the following functions however you like. Explain all steps.

(1) $f(x) = x^3 e^x$

(2) $f(x) = \frac{\sin x}{x^2}$

(3) $f(x) = (4x^3 + x)^6$

$$(4) f(x) = \ln(\cos x)$$

$$(5) f(x) = \cos(\ln x)$$

Problem 8. (10 points) Let $P : y = x^2$ be the standard parabola in the xy plane. Let E_c be the family of ellipses $E_c : 2y^2 + x^2 = c$ (for $c > 0$). Sketch and label a graph showing P , E_9 , and E_{25} . Also, show that every ellipse E_c is orthogonal to the parabola P . (Hint: you do *not* need to explicitly compute the intersection points of P and E_c to solve this problem.)