Math 151: Sections 1-3, 74.
Second Midterm.

Name:____

Discussion section meeting time:______________________

Print your name and discussion section time in the space provided above. Please turn off all cell phones, pagers, music devices, etc. You may not use notes, books, or calculators. (You may use a ruler.) Read each problem carefully. When your final answer is a number or a formula, please draw a box around it. Correct answers without justification will receive no credit. Write neatly: we cannot grade illegible papers. Partial credit will be given for incomplete answers which display some understanding of the problem.

There are 8 problems. You have 80 minutes to work. Good luck!

Problem	1	2	3	4	5	6	7	8	Total
Poss.	10	10	15	10	15	15	10	15	100
Score									

Problem 1 (10 points)**.** Find the exact value of each of the following expressions: $e^{3\ln 2}$ and $\tan(\arcsin(\sqrt{3}/2))$.

Problem 2 (10 points)**.** For what value of r does the function $y = e^{rx}$ satisfy the differential equation $y'' + 5y' - 6y = 0$?

Problem 3 (15 points)**.** Find the first and second derivatives of the function $f(x) = x^2e^{-x}$. Use this information to sketch the curve. Label all roots, extrema, inflection points, and asymptotes. Find all intervals where f is increasing, decreasing, concave up, or concave down.

Problem 4 (10 points)**.** Use a linear approximation to estimate $\ln(1.07)$.

Problem 5 (15 points)**.** Explain why Newton's method doesn't find any root of the equation $x^3 - 3x = 0$ if the initial guess is chosen to be $x_0 = 1$. Illustrate your explanation with a rough sketch of the graph of $x^3 - 3x$.

Problem 6 (5 points each)**.** Compute the following limits. You may use any tools you wish but explain all of your steps.

(1) $\lim_{x\to 1} \frac{x^9-1}{x^5-1}$.

(2) $\lim_{x\to 0} \frac{e^x-1-x}{x^2}$.

(3) $\lim_{x\to \infty} \frac{\sqrt{x^2+x}}{x}$.

Problem 7 (10 points)**.** Find two positive numbers whose product is 100 and whose sum is a minimum. Explain all steps.

Problem 8 (15 points)**.** We are asked to construct a race track of total length exactly 400 yards. The track is to enclose a field which has the following shape: A rectangle with one semicircle attached to each end. (See the figure below.) What is the shape of the track which maximizes the area of the rectangle?