REVIEW PROBLEMS FOR FINAL EXAM
MATH 151, FALL 2004

1. What values of B make the following function continuous at $x = 1$?

$$f(x) = \begin{cases} 3x^3 - x^2 - Bx & x > 1 \\ Bx - 2 & x \leq 1 \end{cases}.$$

2. Find the equation of the tangent line to the curve $y = \dfrac{x-1}{x+1}$ at each point where the tangent is parallel to the line $x - 2y = 2$.

3. Find the equation of the tangent line to the curve defined by the equation $\ln(xy) + 2x - y + 1 = 0$ at the point $(\frac{1}{2}, 2)$.

4. Find the following limits:

a) $\lim_{x\to -1}(x^2 - 2x + 1)$.   b) $\lim_{x\to\infty} \dfrac{3x^2 - 7}{\sqrt{x^2 + 2}}$.   c) $\lim_{x\to 0} \dfrac{\sin 3x}{2\sin 5x}$.

d) $\lim_{x\to\infty} \dfrac{3e^x + 4e^{-x}}{5e^x + 4e^{-x}}$.   e) $\lim_{x\to 8^-} \dfrac{|x-8|}{x-8}$.

5. Find the following limits :

a) $\lim_{x\to 0^+} \sqrt{x}\ln x$.   b) $\lim_{x\to 0^+} x^{\sin x}$.   c) $\lim_{x\to\infty} \dfrac{(\ln x)^2}{x}$.

d) $\lim_{x\to\infty} \dfrac{3e^x + 4e^{-x}}{5e^x + 4e^{-x}}$.

6. Find y' in each case. Do not simplify your answer.

a) $y = x^7 - 3x + 6 - \dfrac{1}{x^4}$.   b) $y = x^{5x}\sin(x^2)$.   c) $y = \dfrac{2\tan x}{\sqrt{1-x^2}}$.

d) $x^4y + 5y^6x^3 = 8$.   e) $xe^{xy+3y} = y$.   f) $y = (1+2x)^{1/x}$.

7. State the formal definition of the derivative of the function $f(x)$. Use the definition to calculate $f'(x)$ for $f(x) = \sqrt{3-5x}$.

8. Suppose that $S(x) = \sqrt{x}$ for $x \geq 0$. Let f and g be differentiable functions about which the following is known:

$$f(3) = 2, \qquad f'(3) = 7, \qquad g(3) = 4, \qquad g'(3) = 5.$$

Compute the following:

$$(f+g)'(3), \qquad (f\cdot g)'(3), \qquad \left(\frac{f}{g}\right)'(3), \qquad (S\circ g)'(3), \qquad \frac{f\cdot g}{f-g}(3).$$

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9. Suppose $f''(x) = -3x + \cos(\pi x)$, $f(1) = 2$, and $f'(1) = -1$. What is $f(5)$?

10. A farmer with 450 feet of fencing wants to enclose the four sides of a rectangular region and then divide the region into four pens with fencing parallel to one side of the rectangle. What is the largest possible total area of the four pens?

11. A ladder (with length 13 ft) is leaning against a wall when its base begins to slide along the floor, away from the wall. By the time the base is 12 ft away from the wall, the base is moving at the rate of 5 ft/sec. How fast is the top of the ladder sliding down the wall then? How fast is the area of the triangle formed by ladder, wall, and floor changing at that time?

12. Let $f(x) = \dfrac{3x}{x^2 - 1}$. Find the domain of the function, the intervals where $f(x)$ is increasing or decreasing, maximum and minimum, the concavity and inflection points, horizontal and vertical asymptotes of the graph of $f(x)$. Then sketch the graph of $f(x)$.

13. Repeat problem **12** for $y = \dfrac{x^2}{x^2 + 3}$.

14. Sketch the graph of $f(x)$ which satisfies the following conditions: $f'(1) = f'(-1) = 0$, $f'(x) < 0$ if $|x| < 1$, $f'(x) > 0$ if $1 < |x| < 2$, $f'(x) = -1$ if $|x| > 2$, $f''(x) < 0$ if $-2 < x < 0$, inflection point $(0, 1)$.

15. Find the linearization of $f(x) = \sqrt{x+1}$ at $a = 15$ and use it to find an approximation to $\sqrt{15}$ and to $\sqrt{17}$. Use the second derivative to determine whether the estimate is greater or less than the actual value.

16. Find the derivative of the following functions:

a) $F(x) = \displaystyle\int_{\pi}^{x} \tan(s^2)ds$. b) $g(x) = \displaystyle\int_{1}^{\cos x} \sqrt{1 - t^2}dt$.

17. Suppose that f is continuous on $[0, 4]$, $f(0) = 1$, and $2 \leq f'(x) \leq 5$ for all $x \in (0, 4)$. Show that $9 \leq f(4) \leq 21$.

18. Let $3x^3 - 2x^2 + x - 1$. Show that $f(x)$ must have a real root in $[0, 1]$.

19. Show that the equation $x^{101} + x^{51} + x - 1 = 0$ has exactly one real root.

20. Use Newton's method to approximate the root of the equation, $x^4 + x - 4 = 0$ in the interval $[2, 3]$ starting with $x_1 = 2$ and for $n = 3$.

21. If a stone is thrown vertically upward from the surface of the moon with a velocity of 10 m/s, its height (in meters) after t seconds is $h(t) = 10t - t^2$.

(a) What is the velocity of the stone after 3 s?

(b) What is the maximal height of the stone?

22. Find the absolute maximum and absolute minimum values of $f(x) = \dfrac{x}{x^2+1}$ in $[0, 2]$.

23. Find the most general antiderivative of the functions:

a) $f(x) = 1 - x^3 + 5x^5 - 3x^7$. b) $g(x) = \dfrac{5 - 4x^3 + 3x^6}{x^6}$.

c) $f(x) = 3e^x + 7\sec^2 x + 5(1 - x^2)^{-\frac{1}{2}}$.

24. Evaluate the integral, if it exists.

a) $\displaystyle\int_1^9 \frac{\sqrt{u} - 2u^2}{u} du$. b) $\displaystyle\int_0^2 y^2\sqrt{1 + y^3} dy$. c) $\displaystyle\int_1^5 \frac{dt}{(t-4)^2}$.

d) $\displaystyle\int_0^1 t^2\cos(t^3) dt$. e) $\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$.

25. If $\displaystyle\int_1^5 f(x)dx = 12$ and $\displaystyle\int_4^5 f(x)dx = 3.6$, find $\displaystyle\int_1^4 f(x)dx$.

26. Find the area bounded by two curves:

a) $y_1 = 2x^2$ and $y_2 = 8x$. b) $y_1 = \sin x$ and $y_2 = \cos x$, $0 \le x \le \dfrac{\pi}{2}$.

27. Evaluate the limit $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \sin\left(\frac{i\pi}{n}\right)\frac{\pi}{n}$.

28 a) Using the definition of the natural logarithm as an integral, compare areas to prove that $\ln 2 < 1 < \ln 3$.

b) Use a) to deduce that $2 < e < 3$.