Problem 1.1. Fix attention on the implicit lines (in $\mathbb{R}^2$) $A : x - y = 1$ and $B : (1+\sqrt{3})x-(1-\sqrt{3})y = 1$. Give a sketch. Express both of these in the form $\mathbf{n}\cdot(x, y) = C$ where $\mathbf{n}$ is the *normal vector* and $C \in \mathbb{R}$ is a scalar.

Use these new implicit descriptions to compute the angle between the lines. (You do not need to find the point of intersection in order to compute the angle.)

Problem 1.2. Fix attention on the planes $P : z = 1$ and $Q : y = 1$. Sketch P, Q, and $L = P \cap Q$, the line of intersection. Describing L as "the intersection of P and Q" is an implicit description. Describe L explicitly by giving a parametrization $L : t \mapsto (f(t), g(t), h(t))$.

Now, if you rotate yourself to look *along* the line L the "dimensionality" of the situation appears to reduce by one: L looks like a point and P and Q look like lines. Compute the angle between these "lines": this is called the *dihedral angle* between P and Q. (If you need a hint: look at a cube.)

Problem 1.3. As in Problem 1.2, but take planes $P' : x+y+z = 1$ and $Q' : x+y-z = 1$. Again, sketch P', Q', and $L' = P' \cap Q'$. Parametrize $L' : t \mapsto (f'(t), g'(t), h'(t))$. It will help if you first find the normal vectors to P' and Q'. What is their cross product?

Compute *both* dihedral angles between P and Q. (If you need a hint: look at an octahedron.)

As a bit of a challenge: can you compute the dihedral angles of the tetrahedron?

Problem 1.4. We proceed in several steps:

1. Sketch a picture of the *cylinder* $C : x^2 + y^2 = 1$ in $\mathbb{R}^3$. Demonstrate that C is "made-up" of straight-lines. (This shows that C is a *ruled surface*: generated by the motions of a line. See problem 47 page 838 of the book for another use of ruled surfaces.)

2. Sketch a picture of the plane $P : z = -x + 2$.

3. The intersection of C and P is a curve in three-space. We'll call it E. Draw C and P in the same coordinate system to give a picture of E. Now sketch E, all by itself. Describe the *projections* (or *shadows*) of E on the xy, yz, and zx coordinate planes. (Two of the three are very easy!)

4. Describing E as the intersection of C and P is an *implicit* description. Find an *explicit* description of E of the form $\theta \mapsto (f(\theta), g(\theta), h(\theta))$.

One use of this problem is the following: Take a cardboard tube. Cut it along a plane. (*Don't* flatten it and cut along a straight line – that will give something quite different.) Now cut the tube along a ruling line, unroll the tube, and flatten it out. What do you see?