Problem 3.1. A function $u(x, y)$ is *harmonic* if it satisfies the *Laplace equation*: $u_{xx} + u_{yy} = 0$. The left hand side is sometimes represented with the notation Δu.

- For each of the following find all values of k making u harmonic:
  - $u(x, y) = x^2 + ky^2$
  - $u(x, y) = x^3 + kxy^2$
  - $u(x, y) = e^{5x} \cos(ky)$
- Suppose that u is harmonic and that $u_{xx}(0, 0) \neq 0$. Suppose also that the graph of $z = u(x, 0)$ is concave up at $x = 0$. Show that the graph of $z = u(0, y)$ is concave down. (That is, the surface is *saddle-shaped* at $(0, 0)$.)
- Which examples from above have this property? Sketch the graphs of $z = u(x, 0)$ and $z = u(0, y)$ for these examples.

Problem 3.2. Let $F(x, y) = (x^2 - y^2, 2xy)$. Note that F is a function from $\mathbb{R}^2$ to $\mathbb{R}^2$.

- Compute DF, the total derivative of F.
- Let $F^2(x, y) = F(F(x, y))$. That is, F^2 is the function "F composed with itself." Express F^2 in terms of x and y. Compute DF^2 two ways: first directly and second using the chain rule.

Problem 3.3. Consider the surface $S : xyz = 6$. This is identical to the graph of $z = f(x, y) = \frac{6}{xy}$.

- Sketch the intersection of S with the three planes $x = 1$, $y = 2$, and $z = 3$.
- Sketch a picture of S, restricted to the first octant (all coordinates nonnegative).
- Find an equation for $T_{(1,2,3)}$, the tangent plane to S at the point $(1, 2, 3)$. Compute the x, y, and z intercepts of $T_{(1,2,3)}$. Compute their product.
- Let $g(x, y, z) = xyz$. Compute the gradient vector ∇g. Use this to find $\mathbf{n}$, the normal vector to the surface S at the point $(1, 2, 3)$. What do you notice?
- Find an equation for $T_{(a,b,c)}$, the tangent plane to S at the point (a, b, c) where we assume that $abc = 6$. Compute the product of the x, y, and z intercepts of this plane.

Problem 3.4. Let $P = P(A, B, C, D)$ be the plane in $\mathbb{R}^3$ with equation $Ax + By + Cz + D = 0$.

- Compute $\mathcal{D}(A, B, C, D)$, the distance from the origin $(0, 0, 0)$ to the plane $P(A, B, C, D)$.

- Describe the domain and range of the function $\mathcal{D}$. (Not every point of $\mathbb{R}^4$ describes a *real* plane!)
- If λ is a non-zero number, prove that

  $$\mathcal{D}(\lambda A, \lambda B, \lambda C, \lambda D) = \mathcal{D}(A, B, C, D).$$

  Do this algebraically first, and then explain the result geometrically.
- The graph of $\mathcal{D}$ is a subset of $\mathbb{R}^5$. Sketch the cross-sections of this graph where $B = C = 0$. Sketch the cross-sections of this graph where $C = D = 0$.