Problem 4.1. Compute the two products $A \cdot B$ and $B \cdot A$ where:

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

Remember: you multiply matrices by taking the dot product of the **rows** of the first with the **columns** of the second. Is $A \cdot B = B \cdot A$?

Problem 4.2. Recall that the rotation matrix in $\mathbb{R}^2$ is $R_\theta = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$. Let $v = (a, b)$ be a vector in $\mathbb{R}^2$ based at the origin. We can form a new vector $R_\theta v$ by taking the dot products of the **rows** of R_θ with the **column** of v.

1. Prove algebraically that $|R_\theta v| = |v|$.
2. If $w = (c, d)$ is another vector in $\mathbb{R}^2$ prove algebraically that the angle between v and w equals the angle between $R_\theta v$ and $R_\theta w$.
3. Suppose τ is another angle. Prove algebraically that $R_\theta \cdot R_\tau = R_{\theta+\tau}$.

Now explain *geometrically* why these three facts are true. Draw pictures.

Problem 4.3. Consider the function $f(x, y) = x^3 + y^3 - \frac{3}{2}(x^2 + y^2)$.

1. Graph the function with Maple. (It is possible to draw this by hand – think about the cross-sections parallel to the yz plane.)
2. Compute $\nabla f = (f_x, f_y)$, the gradient of f. Find all points where $\nabla f = (0, 0)$. (That is, locate all *critical points* of f.)
3. What does the graph of the function look like at those places? What does the tangent plane doing? Explain.
4. The *Hessian* of f is the matrix of second partial derivatives:

$$H_f = \begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix}.$$

   Compute $\det(H_f)$, the *determinant* of H_f, at the points you found in part 2. Check that your results agree with the "Second Derivatives Test" on pages 954-5 of Stewart.

Problem 4.4. Here is the definition of the *pyramid* $P(p, T)$: choose a triangle T in the xy plane (the *base*) and a point $p \in \mathbb{R}^3$ (the *tip*). The pyramid is the union of all line segments connecting p to a point of T.

1. To get used to the definition sketch a picture of the pyramid $P(p, T)$ where T has vertices $(0, 0, 0)$, $(1, 0, 0)$ and $(0, 1, 0)$ and $p = (0, 0, 1)$.

2. Now we return to the general case. Suppose that T is some triangle in the xy plane with area $A = A(T)$. Suppose that the z-coordinate of p is the number h. Compute, via a one-variable integral, the volume of $P(p, T)$.

3. Now do problem 54 on page 963.