Please *write* this week's workshop with at most one other person. Be sure to acknowledge any and all help you receive with your write-up. As usual, you can *think* with other people but you should not *write* with more than one other person. Please write your answer in complete English sentences. Helpful figures are always welcome.

**Problem 7.1.** (Found on a exam review of Prof. Greenfield's.) Evaluate the line integral $\int_C (x - 2y^2)\, dy$ where C is the arc of the parabola $y = x^2$ with $-2 \le x \le 1$. Give a picture.

**Problem 7.2.** (Likewise.) Evaluate the integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x, y) = \langle x^2y^3, -y\sqrt{x}\rangle$ and $\mathbf{r}(t) = (t^2, -t^3)$ for $0 \le t \le 1$.

**Problem 7.3.** (Likewise.) The constraint $x^4 + x^2y^2 + 2y^4 + z^4 = 1$ defines a closed and bounded set in $\mathbb{R}^3$ and thus the function $f(x, y, z) = xyz$ attains a maximum value on that set. What is this maximum value? Be sure to analyze carefully and completely any system of equations you solve.

**Problem 7.4.** We say that a vector field $\mathbf{F}(x, y) = \langle P(x, y), Q(x, y)\rangle$ is *conservative* if $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$ for all closed plane curves $C \colon [a, b] \to \mathbb{R}^2$. (A plane curve $C$ is *closed* if $C(a) = C(b)$.)

- Show that the *tangential* field $\mathbf{F}(x, y) = \langle -y, x\rangle$ is not conservative by finding a closed curve $C$, explicitly computing $\int_C \mathbf{F}$, and noting that the integral is not zero.
- Show that the *constant* field $\mathbf{G}(x, y) = \langle 1, 0\rangle$ is conservative. What is the integral of $\int_C \mathbf{G}$ if $C$ is not closed? (Say, if $C$ begins at the origin and ends at the point $(1, 0)$.)
- Decide whether or not the *radial* field $\mathbf{H}(x, y) = \langle x, y\rangle$ is conservative or not. Explain your answer.

In addition, given a sketch of each of the above fields.