Please *write* this week's workshop with at most one other person. Be sure to acknowledge any and all help you receive with your write-up. As usual, you can *think* with other people but you should not *write* with more than one other person. Please write your answer in complete English sentences. Helpful figures are always welcome.

Recall Stokes' Theorem:

$$\int_{\partial D} \omega = \int_D d\omega.$$

It follows that there are two easy ways for an integral to vanish. That is, if asked to integrate the left hand side then check if $d\omega = 0$. If asked to integrate the right hand side then check if $\partial D = \emptyset$.

Problem 9.1. Compute the integral $\int_S x\, dydz + 2y\, dxdz + z\, dxdy$ where S is the unit sphere in $\mathbb{R}^3$.

Problem 9.2. Compute the integral $\int_C yz\, dx + xz\, dy + xy\, dz$ where C is the intersection of the plane $P : x + y + z = 0$ and the cylinder $Y : x^2 + y^2 = 1$ in $\mathbb{R}^3$.

Three of the following problems (on variants of Green's Theorem) were shamelessly stolen from Prof. Teixeira's second practice midterm.

Problem 9.3. Find a positively oriented simple closed curve C in $\mathbb{R}^2$ which maximizes the line integral

$$\int_C (y^3 - y)\, dx - 2x^3\, dy.$$

Give a sketch.

Problem 9.4. Let $\mathbf{F} = \frac{\langle -y, x\rangle}{x^2+y^2}$. Sketch $\mathbf{F}$. Show that this field is closed (curl $\mathbf{F} = 0$). Is the field conservative in $\mathbb{R}^2 - \mathbf{0}$? Is it exact? Why does this not contradict the fundamental theorem of line integrals? Explain. (For further exploration, you should compare the behaviour of $\mathbf{F}$ with the behaviour of the field $\mathbf{G} = \langle -y, x\rangle$ or, more generally, with the field $\frac{\langle -y, x\rangle}{(x^2+y^2)^n}$ for various exponents n. Which of these is closed?)

Problem 9.5. Let $\mathbf{F}$ be a closed vector field on $\mathbb{R}^2 - \mathbb{B}$. That is, $\mathbf{F}$ is defined on the set $\{p \in \mathbb{R}^2$ such that $|p| > 1\}$. Suppose that

$$\lim_{|p|\to\infty} |p|\mathbf{F}(p) = 0.$$

Show that $\mathbf{F}$ is conservative in $\mathbb{R}^2 - \mathbb{B}$: for any simple closed curve in $\mathbb{R}^2 - \mathbb{B}$ the line integral $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$. (Hint: there are two kinds of simple closed curve in $\mathbb{R}^2 - \mathbb{B}$.) Give a sketch.

Problem 9.6. Let $\mathbf{F}$ be a vector field defined on $\mathbb{R}^3$. Suppose that there is a constant $\lambda \in \mathbb{R}$ so that

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \lambda$$

for every simple closed curve C. Show that $\mathbf{F}$ is conservative on $\mathbb{R}^3$. Give a sketch.