Math 311: Section 3.

Workshop 1: Writing proofs.

Recall that $\mathbb{N}$ is the set of natural numbers: $\mathbb{N} = \{1, 2, 3, 4, 5, \ldots\}$. Here is the *Principle of induction*:

Principle 1.1. Suppose that $P(n)$ is a statement, for every n in $\mathbb{N}$. Suppose also that $P(1)$ is true and, for every n in $\mathbb{N}$, we have that $P(n)$ implies $P(n+1)$. It follows that $P(n)$ is true for every n in $\mathbb{N}$.

The book gives a slightly different version on page 10.

Problem 1.2. Show that the principle of induction is equivalent to the *well-ordering* of the natural numbers: Every non-empty subset $A \subset \mathbb{N}$ has a least element.

Problem 1.3. Give a proof that the number $\sqrt{n}$, for n in $\mathbb{N}$, is *constructible*: there is a finite ruler and compass construction which, beginning with a line segment of length one, produces a line segment of length $\sqrt{n}$. You may use the principle of induction, if you like. Can you find a direct proof?

As a note: a figure, or sequence of figures, illustrating your proof, would be most welcome. However, pictures should not be relied upon to replace a clear exposition.

Problem 1.4. Prove that, for all n in $\mathbb{N}$, the number $\sqrt{n}$ is either a natural number or is an irrational number. (That is, $\sqrt{n}$ is never equal to p/q where p and q are in $\mathbb{N}$, $q > 1$, and $\gcd(p, q) = 1$.)