Math 311: Section 3.

Workshop 2: The field axioms.

A few notes on writing up workshops – if you work with somebody on a problem you must give credit where credit is due. Also, while it is encouraged to work with other students, in class or out of class, all written work turned in for grading must be written by you alone. A fine way to collaborate is to get together, think about the problem, and then separate in order to write up solutions on your own.

Problem 2.1. We say a real number $x \in \mathbb{R}$ is *positive* if $0 < x$. Using only the field axioms, prove that 1 is positive. (Hint: show that squares of non-zero numbers are positive. Another hint: Show that if x is not zero then either x is positive or $-x$ is.)

Problem 2.2. Prove that the set $\mathbb{Q} \cdot \sqrt{2} = \{\frac{p\sqrt{2}}{q} \mid p \in \mathbb{Z}, q \in \mathbb{N}\}$ is dense in $\mathbb{R}$. Now use this fact to deduce Corollary 1.4.4 of the book. (Hint: what is the intersection of $\mathbb{Q}$ with $\mathbb{Q} \cdot \sqrt{2}$?)