**Math 311: Section 3.**

**Workshop 4: Cauchy condesation.**

**Problem 4.1.** Define

$$\zeta(x) = \sum_{n=1}^{\infty} \frac{1}{n^x}.$$

This is the *Riemann zeta function*. Show that $\zeta(x)$ converges if and only if $x > 1$. (This is Corollary 2.4.7 of Abbott's book, as well as Exercise 2.7.7.) Here is a sketch of a proof: you are to provide the details.

(1) Provide the missing direction of the Cauchy Condensation Test, stated as Theorem 2.4.6.
(2) Apply the test to $\zeta(x)$. What series do you obtain?
(3) Use the facts about geometric series given in Example 2.7.5 to decide the convergence of the series obtained in step 2, above. (You may assume without proof that if $y$ is positive and real then $2^y > 1$ while $2^y \in (0, 1]$ if $y \leq 0$.)

**Problem 4.2.** Prove that the *general harmonic series*

$$\sum_{n=0}^{\infty} \frac{1}{a+nb} = \frac{1}{a} + \frac{1}{a+b} + \frac{1}{a+2b} + \frac{1}{a+3b} + \ldots,$$

for $a$ and $b$ positive, diverges.

**Problem 4.3.** Does the series $\sum_{n=2}^{\infty} \frac{1}{n \log_2(n)}$ converge or diverge? Give a complete proof.