Math 311: Section 3.

Workshop 7: What goes around, comes around.

Try to think about as many of these problems as possible. Then turn in any two of them.

Problem 7.1. A hiker, setting out from her car, starts walking up Mt. Analysis at 8am. She gets to the top and pitches a tent at 8pm. (Wotta view!) The next day, again starting at 8am, she walks down the mountain, somewhat surprised to find herself arriving at her car at precisely 8pm.

Show that there was a moment in the first day, call it time t_0, so that at time t_0 and at time $t_0 + 24$hrs the hiker was at the same elevation at both times.

Problem 7.2. Fix a polynomial of odd degree $p(x) = x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_2x^2 + a_1x + a_0$. Show that p has at least one real root. (Give as precise a proof as you can – no hand-waving!)

Extra credit: give upper and lower bounds on the position of the root. Extra-extra credit: compare the bounds you find with *actual* polynomials of odd degree. How tight are your bounds? Can you improve them?

Problem 7.3. Here are two problems which are closely related to each other:

(1) Show that there is no onto continuous function from $\mathbb{R}$ to $\mathbb{R}$ taking on every value exactly twice.
(2) Show that there does exist an onto continuous function from $\mathbb{R}$ to $\mathbb{R}$ taking on every value exactly three times. Can you find an algebraic formula for such a function?

Problem 7.4. Here is problem 4.5.8 from the book: Imagine a clock where the hour hand and the minute hand are indistinguishable from each other. Assuming the hands move continuously around the face of the clock, and assuming their positions can be measured with perfect accuracy, it is always possible to determine the time?