These problems are not to be turned in.

Problem 1.1 (Easy). Let v, w be vectors in $\mathbb{R}^3$, based at the origin. Write down the definition of the norm, $|v|$, in coordinates. Write down the definition of the dot product $v \cdot w$, also in coordinates. Now do enough algebra to prove

$$|v + w|^2 = |v|^2 + |w|^2 + 2v \cdot w.$$

Problem 1.2 (Hard). Suppose that $u = (u_1, u_2, u_3)$, $v = (v_1, v_2, v_3), w = (w_1, w_2, w_3)$ are vectors in $\mathbb{R}^3$. Recall that the *parallelepiped* spanned by these vectors is the set

$$P(u, v, w) = \{au + bv + cw \mid a, b, c \in [0, 1]\}.$$

Define $V(u, v, w)$ to be the *signed volume* of $P(u, v, w)$. That is: if u, v, w in that order obey the right hand rule then V is equal to the volume. On the other hand, if u, v, w obey the left hand rule then V is equal to the negative of the volume.

Working from first principles, give a formula for $V(u, v, w)$.

Remark (Problem solving). If you ever get stuck working on a problem (for example Problem 1.2 above) you might try to:

1. Draw a picture.
2. Exploit a symmetry.
3. Solve an easier, related problem.
4. Explain your ideas to someone.
5. Show that the problem has no solution.