Problem 2.1 (Easy). Let u, v, w be vectors in $\mathbb{R}^3$, based at the origin. The *tetrahedron* spanned by these vectors is the set

$$T(u, v, w) = \{au + bv + cw \mid a, b, c \geq 0,\ a + b + c \leq 1\}.$$

Give a careful, labelled sketch of T. Compute the volume of T. Compute the surface area of T (the sum of the areas of the four faces).

Problem 2.2 (Medium). Fix p and q, points in $\mathbb{R}^3$, as well as v and w, vectors based at p and q respectively. Assume that v and w are *not* parallel. Let $L(s) = p + s \cdot v$ and $M(t) = q + t \cdot w$ be the lines through p, q in the directions v, w. Since v and w are not parallel L and M are called *skew lines*.

Find a formula for the distance between the lines L and M. Where did you use the fact that the lines are skew?

Problem 2.3 (Explore). (Stolen from Prof. Greenfield.) Suppose that u and v are vectors in $\mathbb{R}^3$. Consider the sequence of vectors $\{W_n\}$ defined recursively by the following conditions:

$$W_0 = u, \quad W_{n+1} = W_n \times v \quad \text{for } n > 0.$$

So, for example, $W_3 = ((u \times v) \times v) \times v$.

1. Can you choose u and v so that the sequence eventually lies in a plane in $\mathbb{R}^3$? If your answer is "yes" give the example and explain why it works. If your answer is "no" explain why.

2. Can you choose u and v so that the sequence $\{W_n\}$ has a limit in $\mathbb{R}^3$? If your answer is "yes" give an example and verify it works. If your answer is "no" explain why. (Try some experiments. Interesting examples are better than simple examples.)

3. Can you choose u and v so that the sequence $\{W_n\}$ is eventually periodic? If your answer is "yes" give an example and verify it works. If your answer is "no" explain why. (Again, interesting examples are better than simple examples.)