Problem 3.1. Fix attention on the implicit lines (in $\mathbb{R}^2$) $A : x - y = 1$ and $B : (1+\sqrt{3})x-(1-\sqrt{3})y = 1$. Give a sketch. Express both of these in the form $\mathbf{n}\cdot(x, y) = C$ where $\mathbf{n}$ is the *normal vector* and $C \in \mathbb{R}$ is a scalar.

Use these new implicit descriptions to compute the angle between the lines. (You do not need to find the point of intersection in order to compute the angle.) Draw a picture.

Problem 3.2. Fix attention on the planes $P : z = 1$ and $Q : y = 1$. Sketch P, Q, and $L = P \cap Q$, the line of intersection. Describing L as "the intersection of P and Q" is an implicit description. Describe L explicitly by giving a parametrization $L : t \mapsto (f(t), g(t), h(t))$.

Now, if you rotate yourself to look *along* the line L the "dimensionality" of the situation appears to reduce by one: L looks like a point and P and Q look like lines. Compute the angle between these "lines": this is called the *dihedral angle* between P and Q. (If you need a hint: look at a cube.)

Problem 3.3. As in Problem 3.2, but take planes $P' : x+y+z = 1$ and $Q' : x+y-z = 1$. Again, sketch P', Q', and $L' = P' \cap Q'$. Parametrize $L' : t \mapsto (f'(t), g'(t), h'(t))$. It will help if you first find the normal vectors to P' and Q'. What is their cross product?

Compute *both* dihedral angles between P and Q. (If you need a hint: look at an octahedron.)

Problem 3.4. We proceed in several steps:

1. Sketch a picture of the *cylinder* $C : x^2 + y^2 = 1$ in $\mathbb{R}^3$. Demonstrate that C is "made-up" of straight-lines. (This shows that C is a *ruled surface*: generated by the motions of a line. See problem 47 page 838 of the book for another use of ruled surfaces.)

2. Sketch a picture of the plane $P : z = -x + 2$.

3. The intersection of C and P is a curve in three-space. We'll call it E. Draw C and P in the same coordinate system to give a picture of E. Now sketch E, all by itself. Describe the *projections* (or *shadows*) of E on the xy, yz, and zx coordinate planes. (Two of the three are very easy!)

4. Describing E as the intersection of C and P is an *implicit* description. Find an *explicit* description of E of the form $\theta \mapsto (f(\theta), g(\theta), h(\theta))$.

One use of this problem is the following: Take a cardboard tube. Cut it along a plane. (*Don't* flatten it and cut along a straight line – that will give something quite different.) Now cut the tube along a ruling line, unroll the tube, and flatten it out. What do you see?