Problem 4.1. Suppose that $L(t) = (t, 0, -1)$ and $M(s) = (0, s, 1)$ are two lines in three-space. Let S be the set of points *equidistant* from L and from M: that is, every point $p \in S$ has the *same* distance to both L and to M. Give a sketch of S. Describe S implicitly, as the set of solutions of some equation. What are the cross sections of S by planes of the form $z = c$, $c \in \mathbb{R}$?

Problem 4.2. Consider the map $g(s, t) = (s^2, st, t^2)$ which takes points of the st–plane into the xyz–space. What is the image of this map? Find the cross-sections of the image by planes of the form $z = c$, for $c \in \mathbb{R}$. Give a sketch of the surface.

Problem 4.3. Find the length of the curve $p(t) = (t, \frac{t^2}{\sqrt{2}}, \frac{t^3}{3})$ for $0 \leq t \leq 1$. Give a sketch. (The discussion of arclength in Section 13.3 will be useful.)

Problem 4.4 (Hard). Let $p(t) = (t, t^2, t^3)$ be the *twisted cubic*. Suppose that $a, b, c \in \mathbb{R}$ are distinct real numbers. Show that the points $A = p(a)$, $B = p(b)$, and $C = p(c)$ are not co-linear. (That is, do not lie on a line.)

Problem 4.5 (Very challenging). Show that for any ellipsoid $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ there is a plane P so that

- $(0, 0, 0) \in P$ and
- the cross section $E \cap P$ is a circle.

Conversely, find a quadric surface Q so that, for *any* plane P, the cross section $Q \cap P$ is *not* a circle. Generally, classify the quadric surfaces which do have a round cross section, and those that don't. (Eg, does the elliptic paraboloid have a round cross section? Play around with a computerized drawing program.)