Problem 6.1. Let U be the unit square in $\mathbb{R}^2$: that is, U has vertices at the points $(0,0),(1,0),(1,1),(0,1)$. Consider the transformation $S\colon \mathbb{R}^2 \to \mathbb{R}^2$ defined by $S(x,y) = (x^2 - y^2, 2xy)$.

- Compute DS, the total derivative of S, and find the determinant $\det(DS)$.
- Carefully draw a picture of U. Label the corners and edges. Carefully draw $V = S(U)$, the image of U under S. Label the corresponding corners and edges. Indicate the images of typical horizontal and vertical lines.
- Compute the area of V using one-variable techniques.
- After glancing at page 991 of the book, compute the double integral

$$\iint_U 4(x^2 + y^2)\, dA.$$

Problem 6.2 (Lagrange multiplier problem). Let G be the graph of the function $z = xy$. Sketch G. For every point $P_a = (0,0,a)$ find the distance between P_a and G. (Hint: it is a bit easier to minimize the square of the distance. Hint: if you are having trouble, try fixing $a = 1/10$.)

Extra: Discuss the geometric meaning of all of the non-minimal solutions found by the Lagrange multiplier method.

Problem 6.3. Let U be the rectangle in $\mathbb{R}^2$ with vertices at $(0,0),(2\pi,0),(2\pi,2\pi),(0,2\pi)$. Consider the transformation $E\colon \mathbb{R}^2 \to \mathbb{R}^2$ defined by

$$E(x,y) = (e^{x+y}\cos(y), e^{x+y}\sin(y)).$$

- Compute DE, the total derivative of E, and find the determinant $\det(DE)$.
- Carefully draw and label $V = E(U)$, the image of U under E. Indicate the images of typical horizontal and vertical lines.
- Discuss how a tiny square in U is transformed by E. In particular, how does E distort the area of the tiny square? Sketch an example.
- Compute the area of V.