Problem 7.1 (Lagrange multiplier problem). Let G be the graph of the function $z = x^2 + 2y^2$. Sketch G. Let $P_a = (0, 0, a)$ and find the minimal distance between P_a and G. Explain geometrically the various other solutions given by the method. Why do different values of a lead to different numbers of critical points?

Problem 7.2 (Lagrange multiplier problem). Suppose that, in the problem above, G is the graph of $z = x^2 + y^2$. Draw a picture. What do you expect to happen in this case? How many critical points will there be when a is large and positive? When a is small and positive?

Now solve the problem using Lagrange. At what value of a does the behavior change? Why? Explain geometrically.

Problem 7.3. (See also Problem 36, page 1009 of Stewart.) The goal of this problem is to compute $I = \int_{-\infty}^{\infty} e^{-x^2} dx$. This is defined to be the limit $\lim_{a\to\infty} \int_{-a}^{a} e^{-x^2} dx$. We proceed as follows:

1. Draw a picture of the graph. (This is a version of the famous Bell Curve.)
2. Square I and change the variable of integration of the second copy of I to be y instead of x.
3. Rewrite I^2 as a multivariable integral. Give the definition of this indefinite multivariable integral. (Hint: integrate over squares with sidelength $2a$, centered at the origin. Take a limit.) Draw a picture.
4. Change to polar coordinates. *Briefly* discuss why you can take the limit of the integral over disks of radius a, instead of squares. (That is; argue why the difference of the two integrals goes to zero as $a \to \infty$.) Draw a picture.
5. Find the value of the resulting improper integral by computing the polar integral over a disk of radius a and taking a limit.
6. Take a square root to find the value of I.