Please write in complete English sentences. Helpful figures are always welcome.

Problem 9.1. (Found on a exam review of Prof. Greenfield's.) Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x, y) = \langle x^2 y^3, -y\sqrt{x} \rangle$ and $\mathbf{r}(t) = (t^2, -t^3)$ for $0 \leq t \leq 1$.

Problem 9.2. (Review problem.) We say that a vector field $\mathbf{F}(x, y) = \langle P(x, y), Q(x, y) \rangle$ is *conservative* if $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$ for all closed plane curves $C\colon [a, b] \to \mathbb{R}^2$. (A plane curve C is *closed* if $C(a) = C(b)$.)

- Show that the *tangential* field $\mathbf{F}(x, y) = \langle -y, x \rangle$ is not conservative by finding a closed curve C, explicitly computing $\int_C \mathbf{F} \cdot d\mathbf{r}$, and noting that the integral is not zero.
- Show that the *constant* field $\mathbf{G}(x, y) = \langle 1, 0 \rangle$ is conservative. What is the integral of $\int_C \mathbf{G} \cdot d\mathbf{r}$ if C is not closed? (Say, if C begins at the origin and ends at the point $(1, 0)$.)
- Decide whether or not the *radial* field $\mathbf{H}(x, y) = \langle x, y \rangle$ is conservative or not. Explain your answer.

In addition, given a sketch of each of the above fields.

Problem 9.3. (See p. 1138 of the book.) Find a positively oriented simple closed curve C in $\mathbb{R}^2$ which maximizes the line integral

$$\int_C (y^3 - y)\, dx - 2x^3\, dy.$$

Give a sketch. (Hint: Green's Theorem.)

Problem 9.4. Let $\mathbf{F}$ be a vector field defined on $\mathbb{R}^2$. Suppose that there is a constant $\lambda \in \mathbb{R}$ so that

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \lambda$$

for every closed curve C. Show that $\mathbf{F}$ is conservative. Give a sketch.