Feel free to work with other students on these problems. However all written work should be your own. Also, be sure to give written credit, on the assignment, for any ideas you get from other people.

All proofs should be as short and clear as possible. If you deviate from the style of proof given in the notes you should only do so consciously and for good reason.

Exercise 4.1. We defined a binary relation $<_L$ on $\mathbb{N}\times\mathbb{N}$ by

$$\langle a, b\rangle <_L \langle c, d\rangle \iff \text{either } a < c \text{ or } (a = c \text{ and } b < d).$$

The notes proved that $<_L$ is a linear order on $\mathbb{N}\times\mathbb{N}$. Prove that $<_L$ is a well-ordering: every nonempty subset of $\mathbb{N}\times\mathbb{N}$ has a smallest element. (You may use without proof the fact that $\mathbb{N}$ is well-ordered under the usual ordering.)

Exercise 4.2 (Exercise 8, page 78, Enderton). Suppose that A is a set. Let $F\colon A \to A$ be an injective function which is not surjective. Fix $a \in A \setminus \operatorname{ran} F$. Define $h\colon \omega \to A$ by recursion:

$$\begin{aligned} h(0) &= a \\ h(n^+) &= F(h(n)) \end{aligned}$$

Prove that h is injective.

Exercise 4.3. Show that $m \cdot 1 = m$. (Hint: there is a four line proof.)