

Feel free to work with other students on these problems. However all written work should be your own. Also, be sure to give written credit, on the assignment, for any ideas you get from other people.

All proofs should be as short and clear as possible. If you deviate from the style of proof given in the notes you should only do so consciously and for good reason.

Exercise 5.1. Prove that for all $m \in \omega$, $m = \emptyset$ or $\emptyset \in m$. (Hint: Argue by induction on m .)

Exercise 5.2 (Easy). Let $<$ be a linear order on a set A . If $a, b \in A$ satisfy $a \leq b$ and $b \leq a$ then $a = b$.

Exercise 5.3. Suppose that $f: A \rightarrow B$ is order-preserving. Then the following statements are true.

- If $a_1, a_2 \in A$, then $a_1 <_A a_2$ iff $f(a_1) <_B f(a_2)$.
- f is an injection.
- If f is a bijection, then $f^{-1}: B \rightarrow A$ is also order-preserving.

Exercise 5.4. Suppose that A is a transitive set. Then

- $\mathcal{P}(A)$ is also a transitive set.
- $A \subseteq \mathcal{P}(A)$.

Exercise 5.5. Suppose A, B, C are sets. Carefully and clearly prove:

1. $A^C \cap B^C = (A \cap B)^C$.
2. $A^C \cup B^C \subseteq (A \cup B)^C$.