Feel free to work with other students on these problems. However all written work should be your own. Also, be sure to give written credit, on the assignment, for any ideas you get from other people.

All proofs should be as short and clear as possible. If you deviate from the style of proof given in the notes you should only do so consciously and for good reason.

Exercise 7.1. Which of the following functions from $\mathbb{Z}$ to $\mathbb{Z}$ are well-defined? Explain.

- $F([\langle m,n\rangle]) = [\langle m+n,n\rangle]$.
- $G([\langle m,n\rangle]) = [\langle m,n+5\rangle]$.
- $H([\langle m,n\rangle]) = [\langle m,m\rangle]$.
- $K([\langle m,n\rangle]) = [\langle m^2+n^2,2mn\rangle]$.

Exercise 7.2 (Proposed by Kevin Cantwell). Suppose that $a,b,c \in \mathbb{Z}$ and c is positive. Prove that

$$a <_{\mathbb{Z}} b \quad \text{iff} \quad a \cdot_{\mathbb{Z}} c <_{\mathbb{Z}} b \cdot_{\mathbb{Z}} c.$$

Exercise 7.3 (Enderton, p. 111). Give a direct proof that, for any $r,s \in \mathbb{Q}$, if $r \cdot_{\mathbb{Q}} s = 0_{\mathbb{Q}}$ then $r = 0_{\mathbb{Q}}$ or $s = 0_{\mathbb{Q}}$.