Feel free to work with other students on these problems. However all written work should be your own. Also, be sure to give written credit, on the assignment, for any ideas you get from other people.

All proofs should be as short and clear as possible. If you deviate from the style of proof given in the notes you should only do so consciously and for good reason.

Exercise 3.1. Prove that $\mathbb{R} \smallsetminus \mathbb{N} \sim \mathbb{R}$.

Exercise 3.2. Prove that $\mathbb{R} \smallsetminus \mathbb{Q} \sim \mathbb{R}$.

Exercise 3.3. Let $\mathrm{Sym}(\mathbb{N}) = \{ f \mid f \colon \mathbb{N} \to \mathbb{N} \text{ is a bijection } \}$. Prove that $\mathcal{P}(\mathbb{N}) \sim \mathrm{Sym}(\mathbb{N})$.

Exercise 3.4. Let $A = \{ \langle a, b \rangle \mid a, b \in \mathbb{Z},\ b \neq 0 \}$. Define the relation S on A by

$$\langle a, b \rangle S \langle c, d \rangle \ \text{ iff } \ ad - bc = 0.$$

Prove that S is an equivalence relation. Do not use fractions anywhere in your proof.

Exercise 3.5. How many equivalence relations can be defined on $A = \{1, 2, 3, 4\}$? List them in a reasonable fashion and explain briefly why your list is complete.

Exercise 3.6. Prove that $\langle \mathbb{Z}, < \rangle \not\cong \langle \mathbb{Q}, < \rangle$.