Feel free to work with other students on these problems. However all written work should be your own. Also, be sure to give written credit, on the assignment, for any ideas you get from other people.

All proofs should be as short and clear as possible. If you deviate from the style of proof given in the notes you should only do so consciously and for good reason.

Exercise 4.1. Prove the claim made in class that any dense linear order $\langle A, < \rangle$, with at least two points, is infinite. (Since no assumption about endpoints is made you cannot use Theorem 8.17 from the notes.)

Exercise 4.2. A wff σ is a *tautology* if $\bar{v}(\sigma) = T$ for every truth assignment v. Suppose that A, B, C are sentence symbols. Prove that

$$((A{\wedge}(B{\wedge}C)){\leftrightarrow}((A{\wedge}B){\wedge}C))$$

is a tautology. (That is, "$\wedge$" is associative.)

Exercise 4.3. Show that

$$(((A{\leftrightarrow}B){\wedge}(B{\leftrightarrow}C)){\rightarrow}(A{\leftrightarrow}C))$$

is a tautology but

$$(((A{\leftrightarrow}B){\leftrightarrow}C){\rightarrow}(A{\leftrightarrow}C))$$

is not. (Thus the informal proof technique of writing a chain of "iff"'s is not formally correct.)

Exercise 4.4. Give truth tables for $\sigma = (A{\rightarrow}B)$, for the *contrapositive* $\tau = ((\neg B){\rightarrow}(\neg A))$, for the *converse* $\phi = (B{\rightarrow}A)$, and for the *obverse* $\chi = ((\neg A){\rightarrow}(\neg B))$. Deduce that $(\sigma{\leftrightarrow}\tau)$ is a tautology but $(\sigma{\leftrightarrow}\phi)$ is not.