Feel free to work with other students on these problems. However all written work should be your own. Also, be sure to give written credit, on the assignment, for any ideas you get from other people.

All proofs should be as short and clear as possible. If you deviate from the style of proof given in the notes you should only do so consciously and for good reason.

Exercise 5.1. Consider the following language: the alphabet $\{(,)\}$ contains only two characters, the left and right parentheses. An expression is any sequence of characters. The set of wffs, $\bar{\mathcal{M}}$ in this language is defined recursively by:

1. $() \in \bar{\mathcal{M}}$.
2. If $\alpha \in \bar{\mathcal{M}}$ then so is (α). (Encapsulation.)
3. If $\alpha, \beta \in \bar{\mathcal{M}}$ then so is $(\alpha\beta)$. (Concatenation followed by encapsulation.)

So, for example, $()()$ is *not* a wff in $\bar{\mathcal{M}}$ but $((())())$ is. Count the number of wffs in $\bar{\mathcal{M}}$ with length ten. List them in a reasonable fashion and explain briefly why your list is complete. Challenge: how many wffs of length $2n$ are there?

Exercise 5.2. Suppose that φ is a wff and v_1, v_2 are truth assignments which agree on all sentence symbols appearing in φ. Then $\bar{v}_1(\varphi) = \bar{v}_2(\varphi)$. (*Hint:* argue by induction on the length of φ.)

Exercise 5.3. Let σ, τ be wffs. Then the following statements are equivalent.

1. σ and τ are tautologically equivalent.
2. $(\sigma \leftrightarrow \tau)$ is a tautology.

(Hint: do *not* argue by induction on the lengths of the wffs.)

Exercise 5.4 (Enderton, page 27). Prove or refute each of the following assertions:

1. If either $\Sigma \models \alpha$ or $\Sigma \models \beta$, then $\Sigma \models (\alpha \vee \beta)$.
2. If $\Sigma \models (\alpha \vee \beta)$, then either $\Sigma \models \alpha$ or $\Sigma \models \beta$.