Feel free to work with other students on these problems. However all written work should be your own. Also, be sure to give written credit, on the assignment, for any ideas you get from other people.

All proofs should be as short and clear as possible. If you deviate from the style of proof given in the notes you should only do so consciously and for good reason.

Exercise 6.1. Recall that $A \sim B$ if there is a bijection from A to B. Recall that B^A is the set of functions from A to B. Let 2 be the set $\{0,1\}$ and let 4 be the set $\{0,1,2,3\}$. Prove that $2^{\mathbb{N}} \sim 4^{\mathbb{N}}$.

Exercise 6.2. Let $\{S_n \mid n \in \mathbb{N}\}$ be a collection of finite subsets of $\mathbb{N}$ such that for each finite subset $F \subset \mathbb{N}$, there exists a subset $A_F \subseteq \mathbb{N}$ with $|A_F \cap S_n| = 1$ for all $n \in F$. Prove that there exists a subset $A \subseteq \mathbb{N}$ such that $|A \cap S_n| = 1$ for all $n \in \mathbb{N}$. (Hint: use the Compactness Theorem. Give a "meaning" for your sentence symbols and for the wffs you place in Σ.)

Exercise 6.3. Give an example of a collection $\{T_n \mid n \in \mathbb{N}\}$ of subsets of $\mathbb{N}$ satisfying both of the following conditions.

- For each finite subset $F \subset \mathbb{N}$, there exists a subset $B_F \subseteq \mathbb{N}$ such that $|B_F \cap T_n| = 1$ for all $n \in F$.
- There does *not* exist a subset $B \subseteq \mathbb{N}$ such that $|B \cap T_n| = 1$ for all $n \in \mathbb{N}$.

Exercise 6.4 (Enderton, page 66). When Σ is a set of wffs, define a *deduction* from Σ to be a finite sequence $\langle \alpha_0, \ldots, \alpha_n \rangle$ of wffs such that for each $k \leq n$, either

(a) α_k is a tautology,

(b) $\alpha_k \in \Sigma$, or

(c) for some i and j less than k, $\alpha_i = (\alpha_j \rightarrow \alpha_k)$. (That is, α_k is obtained by *modus ponens* from α_i and α_j.)

If there is a deduction $\langle \alpha_0, \ldots, \alpha_n \rangle$ from Σ then we write $\Sigma \vdash \alpha_n$ and say that α_n can be *deduced* from Σ.

Prove that $\{\alpha, \beta\} \vdash (\alpha \wedge \beta)$.

Exercise 6.5 (Enderton, page 66). Prove that $\{((\neg S) \vee R), (R \rightarrow P), S\} \vdash P$.

Exercise 6.6 (Enderton, page 66). Prove the Soundness Theorem: If $\Sigma \vdash \varphi$ then $\Sigma \models \varphi$. (Hint: Induct on the length of the deduction of φ from Σ.)