Feel free to work with other students on these problems. However all written work should be your own. Also, be sure to give written credit, on the assignment, for any ideas you get from other people.

All proofs should be as short and clear as possible. If you deviate from the style of proof given in the notes you should only do so consciously and for good reason.

Exercise 7.1. We call a wff φ a *contradiction* if $(\neg\varphi)$ is a tautology. (An equivalent definition: φ is a contradiction iff $\bar{v}(\varphi) = F$ for every truth assignment v.) Let Σ be a set of wffs. We call Σ *consistent* if no contradiction can be deduced from Σ. If some contradiction can be deduced from Σ we call Σ *inconsistent*.

Prove the following: If Σ is inconsistent then, for *any* wff τ, $\Sigma \vdash \tau$.

Exercise 7.2. Suppose that Σ is a set of wffs. Use Soundness to prove: If Σ is satisfiable then Σ is consistent.

Exercise 7.3. Prove the "reductio ad absurdum": If $\Sigma \cup \{\neg\tau\}$ is inconsistent then $\Sigma \vdash \tau$.

(Hint: use compactness and the last problem from the last homework set – find a useful contradiction!)

Exercise 7.4. Deduce the Completeness Theorem: If $\Sigma \models \tau$ then $\Sigma \vdash \tau$.