Feel free to work with other students on these problems. However all written work should be your own. Also, be sure to give written credit, on the assignment, for any ideas you get from other people.

All proofs should be as short and clear as possible. If you deviate from the style of proof given in the notes you should only do so consciously and for good reason.

Exercise 8.1. Suppose that $R \subset \mathbb{N}\times\mathbb{N}$ is a relation satisfying the following conditions.

- For all $a \in \mathbb{N}$ the set $\{b \in \mathbb{N} \mid \langle a, b\rangle \in R\}$ is non-empty and finite.
- For every finite subset $A_0 \subseteq \mathbb{N}$, there exists a one-to-one function $f_0\colon A_0 \to \mathbb{N}$ such that $\langle a, f_0(a)\rangle \in R$ for all $a \in A_0$.

Use the Compactness Theorem to prove that there exists a one-to-one function $f\colon \mathbb{N} \to \mathbb{N}$ such that $\langle a, f(a)\rangle \in R$ for all $a \in \mathbb{N}$.

Exercise 8.2. Give an example of a relation $S \subset \mathbb{N}\times\mathbb{N}$ satisfying the following conditions.

- For all $c \in \mathbb{N}$ the set $\{d \in \mathbb{N} \mid \langle c, d\rangle \in S\}$ is non-empty.
- For every finite subset $C_0 \subseteq \mathbb{N}$, there exists a one-to-one function $g_0\colon C_0 \to \mathbb{N}$ such that $\langle c, g_0(c)\rangle \in S$ for all $c \in C_0$.
- There does *not* exist a one-to-one function $g\colon \mathbb{N} \to \mathbb{N}$ such that $\langle c, g(c)\rangle \in S$ for all $c \in \mathbb{N}$.