

Feel free to work with other students on these problems. However all written work should be your own. Also, be sure to give written credit, on the assignment, for any ideas you get from other people.

Exercise 9.1. In a step-by-step process translate the following sentence in the language of set theory into English.

$$\forall v_1 \forall v_2 ((\neg(\forall v_3 (\in v_3 v_1 \rightarrow \in v_3 v_2) \rightarrow (\neg \forall v_3 (\in v_3 v_2 \rightarrow \in v_3 v_1)))) \rightarrow = v_1 v_2)$$

(Hint: you may find it useful to replace v_1 by X , v_2 by Y , and v_3 by z .)

Exercise 9.2. Do the same for:

$$\forall v_1 (\neg \forall v_2 (\neg \forall v_3 (\forall v_4 (\in v_4 v_3 \rightarrow \in v_4 v_1) \rightarrow \in v_3 v_2)))$$

(Hint: Again replace the variables v_i by a more suggestive notation. Explain your notation.)

Exercise 9.3. Give a step by step translation of the following sentence into the first order language of set theory: “For all sets X and Y there is a set Z which contains, as a subset, the union of X and Y .” (Hint: don’t try to define $X \cup Y$.)

Exercise 9.4. Invent an appropriate first order language and, in a step-by-step process, translate the following English sentence into your language: “There is a man living in Smallville and he shaves all the men of Smallville who do not shave themselves.” The only logical symbols allowed in the final product are: $(,)$, \neg , \rightarrow , v_1, v_2, \dots , \forall , and $=$.

(Hint: your language needs at most two non-logical symbols, both predicates.)