

Feel free to work with other students on these problems. However all written work should be your own. Also, be sure to give written credit, on the assignment, for any ideas you get from other people.

All proofs should be as short and clear as possible. If you deviate from the style of proof given in the notes you should only do so consciously and for good reason.

Exercise 10.1. Fix a first order language \mathcal{L} . Let \mathcal{A} be a structure for \mathcal{L} and let t be a term. Suppose that $s_1, s_2: V \rightarrow A$ agree on all variables (if any) in t . Prove that $\bar{s}_1(t) = \bar{s}_2(t)$.

Exercise 10.2. Suppose that $\varphi: A \rightarrow B$ is an isomorphism and $s: V \rightarrow A$. Prove that, for any term t ,

$$\varphi(\bar{s}(t)) = (\overline{\varphi \circ s})(t).$$

Exercise 10.3. Show that the logical axiom

$$(\forall x(\alpha \rightarrow \beta) \rightarrow (\forall x\alpha \rightarrow \forall x\beta))$$

is valid.

Exercise 10.4. Decide whether or not $(Px \rightarrow \forall yPy)$ is valid. Prove your answer is correct.

Exercise 10.5. Prove that $\vdash (\forall xPx \rightarrow \forall yPy)$. That is, provide a deduction of the sentence using only the logical axioms and modus ponens.