Please work in groups of two or three.

Problem 2.1. A few warm-up problems: Let $\mathbf{i} = (1, 0, 0)$, $\mathbf{j} = (0, 1, 0)$, and $\mathbf{k} = (0, 0, 1)$. These are the *unit coordinate vectors*.

1. Compute all of the pairwise dot products: $\mathbf{i} \cdot \mathbf{i}$, $\mathbf{i} \cdot \mathbf{j}$, and so on. There are nine of them. Draw a picture to explain what is going on.
2. Compute all of the pairwise cross products: $\mathbf{i}\times\mathbf{i}$, $\mathbf{i}\times\mathbf{j}$, and so on. Again, there are nine. Draw a picture to explain what is going on.
3. Now get out your right hand. Point your fingers in the direction of $\mathbf{i}$. Now curl your ring finger and pinky in the direction of $\mathbf{j}$. What direction is your thumb pointing? This is called the *right hand rule*. Use the right hand rule to organize your answers to the previous exercise.

   What happens if you use your left hand instead of your right? (The professor is not responsible for orientation-induced injuries.)

Problem 2.2. Another exercise. Sketch the set

$$A := \{(\cos(t), \sin(t)) \in \mathbb{R}^2 \mid 0 \leq t \leq 2\pi/3\}.$$

Problem 2.3. Now that you are warmed up, it's time for a computation! Let $\mathbf{u}$ and $\mathbf{v}$ be vectors in $\mathbb{R}^3$, given their coordinates. Compute, in coordinates, the quantities $|\mathbf{u}|^2 \cdot |\mathbf{v}|^2$, $(\mathbf{u} \cdot \mathbf{v})^2$, and $|\mathbf{u}\times\mathbf{v}|^2$. Verify the coordinate-free indentity

$$|\mathbf{u}|^2 \cdot |\mathbf{v}|^2 = (\mathbf{u} \cdot \mathbf{v})^2 + |\mathbf{u}\times\mathbf{v}|^2.$$

(You will have to organize your algebra. Otherwise it will disorganize *you*.) Now, use the geometric meaning of $\mathbf{u} \cdot \mathbf{v}$ to give a geometric meaning for $\mathbf{u}\times\mathbf{v}$. (Ask for hints if you get stuck.)

Problem 2.4. Suppose that $\mathbf{u}$ and $\mathbf{v}$ are vectors in $\mathbb{R}^3$. Consider the following set of points:

$$P(\mathbf{u}, \mathbf{v}) := \{s\mathbf{u} + t\mathbf{v} \mid 0 \leq s \leq 1,\ 0 \leq t \leq 1\}.$$

Give a sketch of $P(\mathbf{i}, \mathbf{j})$, $P(\mathbf{i}, \mathbf{k})$, and $P(\mathbf{i}, \mathbf{i} + \mathbf{j})$. In each case, compute the area of the set P. How would you find the area of $P(\mathbf{u}, \mathbf{v})$ in general? (Hint: think about the shape of P. Think about the conclusion of the previous problem.)

Problem 2.5. Suppose we are given vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ in $\mathbb{R}^3$. Define

$$P(\mathbf{u}, \mathbf{v}, \mathbf{w}) := \{r\mathbf{u} + s\mathbf{v} + t\mathbf{w} \mid 0 \leq r \leq 1,\ 0 \leq s \leq 1,\ 0 \leq t \leq 1\}.$$

This is a *parallelepiped*. Draw a picture of $P(\mathbf{i}, \mathbf{j}, \mathbf{k})$. Draw a picture of $P(\mathbf{i}, \mathbf{i} + \mathbf{j}, \mathbf{i} + \mathbf{k})$. Compute the volumes.

Now, what does a "general" parallelepiped look like? How would you find its volume?

Problem 2.6 (Hard.). Suppose that T is a triangle with side-lengths a, b, and c. Define the *semiperimeter* $s := (a + b + c)/2$. Give a "vector analysis" proof of Heron's formula:

$$\text{Area}(T) = \sqrt{s(s-a)(s-b)(s-c)}.$$