Please write in complete English sentences. Helpful figures are always welcome.

Stokes' Theorem states that:

$$\int_{\partial D} \omega = \int_D d\omega.$$

It follows that there are two "trivial" ways for an integral to vanish. That is, if asked to integrate the left hand side then check if $d\omega = 0$. If asked to integrate the right hand side then check if $\partial D = \emptyset$.

Problem 11.1. Compute the surface integral $\iint_S x\,dydz + 2y\,dzdx + z\,dxdy$ where S is the unit sphere $x^2 + y^2 + z^2 = 1$ in $\mathbb{R}^3$.

Problem 11.2. Compute the integral $\int_C yz\,dx + xz\,dy + xy\,dz$ where C is the intersection of the plane $P : x + y + z = 0$ and the cylinder $Y : x^2 + y^2 = 1$ in $\mathbb{R}^3$.

Problem 11.3. Let U be the upper hemisphere: the portion of the unit sphere above the xy–plane. Equip U with the upward pointing normal. Let

$$\mathbf{H}(x, y, z) = (z \sin y, z \cos x, \exp(-x^2 - y^2))$$

be a vector field. Compute $\iint_U \mathbf{H} \cdot d\mathbf{S}$.

Problem 11.4. How does the answer change, in problem 11.3, if $\mathbf{H}$ is changed to

$$\mathbf{H}(x, y, z) = (x + z \sin y, z \cos x, \exp(-x^2 - y^2))?$$

Problem 11.5. Suppose that $a > 0$. Prove that

$$\int_0^{2\pi} \frac{a\,dt}{a^2 \cos^2 t + \sin^2 t} = 2\pi.$$

What goes wrong if $a < 0$?