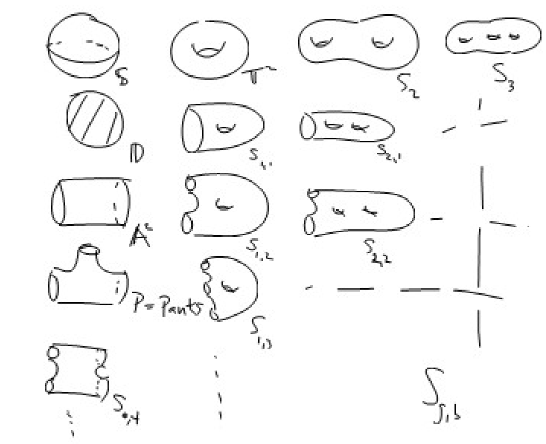
Saul Schleimer

MCG(S) and $\mathcal{C}(S)$

Picture



Outline of class

* MCG(S) = mapping class group
* PML(S) = projectivization of the space of measured laminations
* Teich(S) = Teichmüller space
* $\mathcal{C}(S)$ = complex of curves.

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* Pehn-Lickorish Theorem

  MCG(S) is fin. gen. by twists, half twists, and a reflection.
* MCG acts on PML, T(S), $\mathcal{C}(S)$.
* Thurston compactification of T(S).

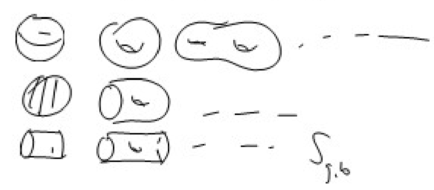
  $$T(S) \cup PML(S) \cong B^{6g-6+2b}$$
* Thurston–Nielsen classification
  * periodic
  * reducible
  * pseudo-Anosov
* Ivanov's Thm

  $$\mathrm{Aut}(\mathcal{C}(S)) \cong MCG(S)$$
* Masur Minsky theory of hierarchies.

  Word length estimate

$$\|g\|_{mcg} \cong \sum_{X \subseteq S}' \left[d_X(\mu, g(\mu)) \right]$$

word length — distance in $\mathcal{C}(X)$ — marking



Thm: (Classification of surfaces)

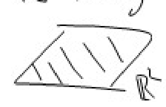
If S, T are connected, compact, orientable surfaces and

$$\text{genus}(S) = \text{genus}(T), \ |\partial S| = |\partial T|$$

Then S, T are homeomorphic.

Surface: 2-manifold, poss with ∂.

i.e. locally $\mathbb{R}^2$ or $\mathbb{R} \times \mathbb{R}_{\geq 0}$



$S^2 = \{ \vec{x} \in \mathbb{R}^3 \mid |\vec{x}| = 1 \}$

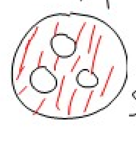
Def: If $S \subseteq \mathbb{R}^2$ call S planar.

Def: If $S \subseteq S^2$ call S planar

Ex: Find homeomorphisms between two kinds of planar. And, fix the mistake

Hint: "stereographic projection"

Exam:

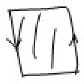


$S_{0,4} \cong \mathbb{D}^2 \setminus 3$ disjoint open disks with disjoint closures.

Connected: Equiv (for us) path connected.

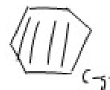
NOT 

Orientable: N. submanifold is homeomorphic to  Möbius band M^2

$M^2 =$  $= I \times I \big/ (0,y) \sim (1,1-y)$

Ex: This is a 2-mfd

Ex: Suppose $P = 2n$-gon  bigon sq.

 6-gon

Use the classification thm to identify all surfaces obtainable as identification space of the $2n$-gon.

Real classification

S^2 $\mathbb{RP}^2 = \mathbb{P}$ $\mathbb{K}^2$	$\mathbb{T}^2$ $3\mathbb{P}$ $4\mathbb{P}$	S_2 $S_{2,0,1}$ $S_{2,0,2}$	
$\mathbb{D}$ $\mathbb{M}$ $\mathbb{K}-\mathbb{D}$	$S_{1,1,0}$ $S_{1,1,1}$ $S_{1,1,2}$	$S_{2,1}$ $S_{2,1,1}$ $S_{2,1,2}$	
$\mathbb{A}$ $M-\mathbb{D}$ $\mathbb{K}-2\mathbb{D}$			

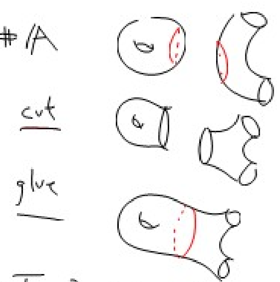
Generally

$$S_{g,b,c} = \#_g \mathbb{T}^2 \# \#_b \mathbb{D}^2 \# \#_c \mathbb{P}^2$$

Exercise: prove $3\mathbb{P} \cong \mathbb{T}^2 \# \mathbb{P}^2$

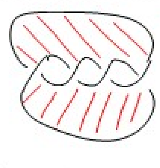
Def: $S \# T$ is a surface obtain by gluing the "new boundary" of $S \cdot \mathbb{D}$ to " " of $T \cdot \mathbb{D}$.

Eg: $\mathbb{T} \# \mathbb{A}$



Ex: $S \# T$ is connected, compact, orient. as S and T were

Homeo: Ie a continuous bijection with continuous inverse.

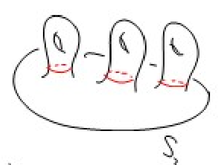
Ex:  Identify this surface in the list given and provide the homeomorphism

[Cut and paste and isotopy in $\mathbb{R}^3$]

Genus: Def 1: The number of holes.

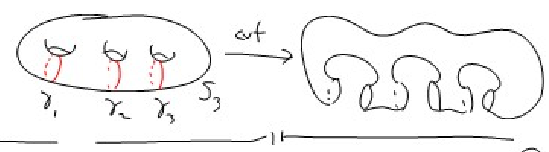
Better: Call $S_{1,1}$ a handle 

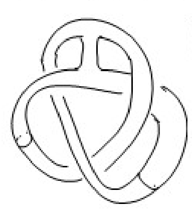
Def 2: genus(S) is the maximal # of disjoint 2-submanifolds $X_1, X_2, \cdots X_g$ in S, which are homeo to $S_{1,1}$

Picture  $g(S_3) \geq 3$.

Q: $g(S_3) \neq 4$.

Def 3: A collection $\Gamma = \{\gamma_1, \gamma_2, \cdots \gamma_g\}$ of disjoint loops is a cut system if $S_\Gamma = S$ cut along Γ is planar.





Compute the genus

Draw the cutsystem the handles.

S.SCHLEIMER @warwick.ac.uk