Suppose X is a simplicial complex with vertex set V.

<u>Def</u>: $\mathrm{Aut}(X) = \left\{ \text{bijections } f: V \to V \;\middle|\; \begin{array}{l} f(\sigma) \in X \text{ iff} \\ \sigma \in X \end{array} \right\}$

Define a map
$$\begin{array}{ccc} \mathrm{MCG}(S) & \longrightarrow & \mathrm{Aut}(\mathcal{C}(S)) \\ \uplus & & \uplus \\ [f] & \longmapsto & ([p] \longmapsto [f(p)]) \end{array}$$

This is well-defined because homeomorphisms preserve disjointness [preserve $i(\cdot,\cdot)$]

<u>Ivanov's Theorem</u>: [Ivanov, Korkmaz, Luo]

$\mathrm{MCG}(S) \longrightarrow \mathrm{Aut}(\mathcal{C}(S))$ is an isomorphism; except for

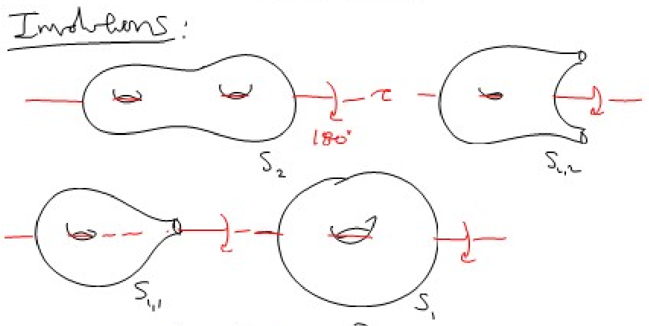
(i) If $S = S_2, S_{1,1}, S_1$ then
$$\mathrm{MCG}(S)/\langle\tau\rangle \cong \mathrm{Aut}(\mathcal{C}(S))$$
[τ is hyperelliptic]

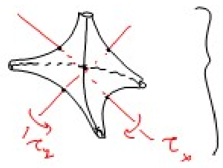
(ii) If $S = S_{1,2}$ then
$$\mathrm{MCG}(S)/\langle\tau\rangle \underset{5}{\leq} \mathrm{Aut}(\mathcal{C}(S))$$

(iii) If $S = S_{0,4}$ then
$$\mathrm{MCG}(S)/\langle\tau_1,\tau_2,\tau_3\rangle \cong \mathrm{Aut}(\mathcal{C}(S))$$

(iv) If $S = S^2, D^2, A, S_{0,3}$ then $\mathcal{C}(S)$ is empty.

<u>Involutions</u>:





Axis of τ_z is perp. to that of τ_x, τ_y.
[Axis connect edge centers]

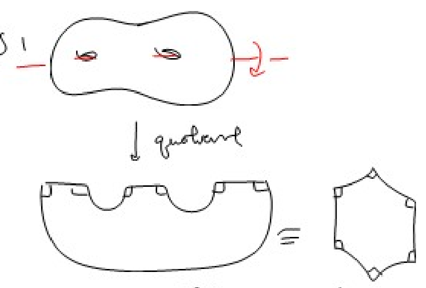
<u>Exercise</u>: In all cases, τ acts trivially on $\mathcal{C}(S)$.

<u>Exercise</u>: Except for τ's, the map
$$\mathrm{MCG}(S) \to \mathrm{Aut}(\mathcal{C}(S)) \text{ is injective.}$$
[Hint: Use Alexander Method]

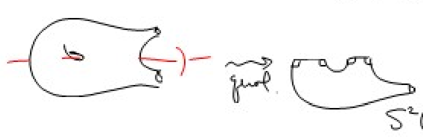
<u>Rmk</u>: In all other cases $Z(\mathrm{MCG}(S))$ (= center) is trivial.

[<u>Idea</u>: If σ is central then $[\sigma, T_\alpha] = \mathbb{1}$ $\forall \alpha$
But $\sigma^{-1} T_\alpha \sigma = T_\alpha$ and
$\sigma^{-1} T_\alpha \sigma = T_{\sigma^{-1}(\alpha)}$ [True for all $f \in \mathrm{MCG}(S)$]]

<u>Pictures</u>:

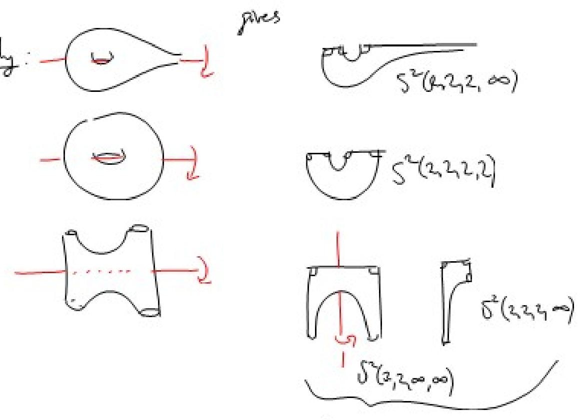


$S^2(2,2,2,2,2,2) \rightsquigarrow S_{0,6}$



$S^2(2,2,2,2,\infty) \rightsquigarrow S_{0,5}$

Lastly:



All equiv to $S_{0,4}$.

Exercise: Show that

$$\mathcal{C}(S_2) \cong \mathcal{C}(S_{0,6}); \quad \mathcal{C}(S_{1,2}) \cong \mathcal{C}(S_{0,5}).$$

In fact Thm: If $\mathcal{C}(S) \cong \mathcal{C}(\Sigma_1)$ then either

(i) $S \cong \Sigma_1$ (homeo)

(ii) $\{S, \Sigma_1\} = \{S_2, S_{0,6}\}$

(iii) $= \{S_{1,2}, S_{0,5}\}$

(iv) $\subseteq \{S_{1,1}, S_1, S_{0,4}\}$

$\subseteq \{S^2, D, A, S_{0,3}\}$ $\boxed{\text{Pf. later}}$

The Farey Tesselation

$\mathcal{F}$ has vertices $\hat{\mathbb{Q}}$ $(\subseteq \mathcal{S}(T))$

and $\left\{\frac{p_i}{q_i}\right\}_{i=0}^{k}$ is a simplex iff

$$\left|\det\begin{pmatrix} p_i & p_j \\ q_i & q_j \end{pmatrix}\right| = 1 \quad \forall i \neq j.$$

Exercises (i) If $\sigma \in \mathcal{F}$, $\dim(\sigma) \leq 2$

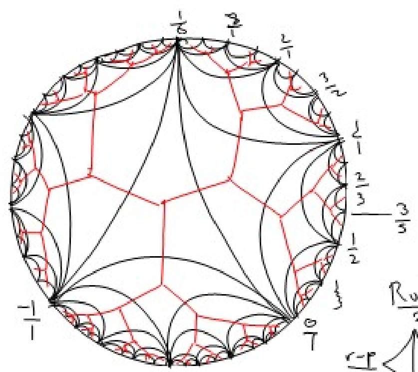
(ii) $\forall \sigma \in \mathcal{F}\ \exists \tau \in \mathcal{F}$ s.t. $\sigma \subseteq \tau$, $\dim \tau = 2$

(iii) Every edge meets exactly 2 triangles

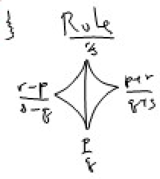
(iv) $\mathcal{F}$ is connected.

(v) Every edge separates $\mathcal{F}$.

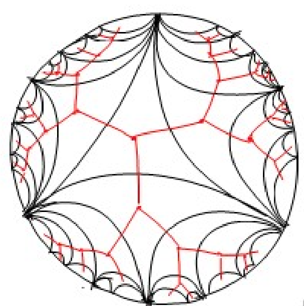
Picture



Rule



Very Harmonious!



Very Hyperbolic!

Recall: $\alpha, \beta \subseteq T^2$ are dual if $i(\alpha,\beta)=1$
$\subseteq S_{1,1}$

So, Edges of $\mathcal{F}$ record dual pairs.

Lemma: $\varphi \in \mathrm{Aut}(\mathcal{F})$ determined by
$\varphi(0), \varphi(\infty), \varphi(1)$.

Lemma: $\mathrm{Aut}(\mathcal{F}) \cong \mathrm{PGL}(2,\mathbb{Z})$.