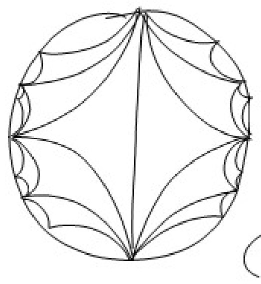


Exercise: Use computer to draw a more accurate picture

(or use compass and straightedge.)

Lemma: $\mathrm{Aut}(\mathcal{F}) \cong \mathbb{P}GL(2,\mathbb{Z})$

$\left[GL(2,\mathbb{Z}) / \pm Id \right]$

Pf:

$$\begin{array}{ccc} GL & \longrightarrow & \mathrm{Aut}(\mathcal{F}) \\ \Psi & & \Psi \\ A & \longmapsto & f_A \end{array} \left. \vphantom{\begin{array}{c} a\\b\\c \end{array}} \right\} \quad A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \quad f_A(p/q) = \frac{cq+dp}{aq+bp}.$$

Since $\det A\begin{pmatrix} p & r \\ q & s \end{pmatrix} = \pm \det \begin{pmatrix} p & r \\ q & s \end{pmatrix}$ this is well defined. **Notice:** $\pm Id \leq$ kernel.

Exercise: $\pm Id =$ kernel. Now we must prove surjectivity. [will spend time on this b/c proof is a model for pf of Ivanov.]

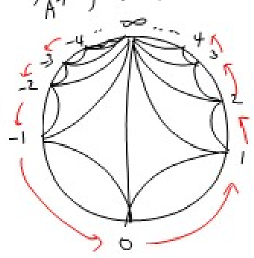
$\boxed{\text{Step 1:}}$ fix $f \in \mathrm{Aut}(\mathcal{F})$. Suppose $f(\infty) = p/q$

choose r, s s.t. $A = \begin{pmatrix} s & q \\ r & p \end{pmatrix} \in GL(2,\mathbb{Z})$

[possible because $\gcd(p,q) = 1$]

So $f_{A^{-1}} \cdot f(\infty) = \infty$. So ∞ now fixed.

Note $\boxed{\text{Step 2}}$



$f_{\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}}$ fixes ∞

and $f_{\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}}(n) = n+1$ [check]

So: $\exists k \in \mathbb{Z}$ s.t. $f^k_{\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}} \cdot f_{A^{-1}} \cdot f$ fixes ∞ and 0.

This because $\{0, \infty\} \in \mathcal{F} \Rightarrow f_{A^{-1}} \cdot f(0)$ is adjacent to ∞, i.e. $f_{A^{-1}} \cdot f(0) \in \mathbb{Z}$.

$\boxed{\text{Step 3}}$ $f^k_{\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}} \cdot f_{A^{-1}} \cdot f$ fixes $\{\pm 1\}$

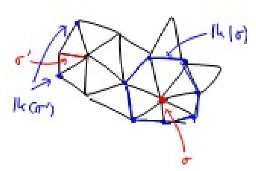
Let $g = Id$ or $f_{\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}}$ so that

$\phi = g \cdot f^k_{\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}} \cdot f_{A^{-1}} \cdot f$ fixes $\{-1, 0, \infty\}$ pointwise.

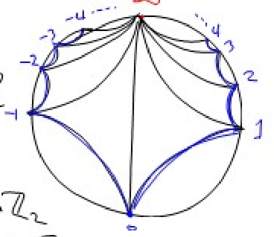
Def: X a simplicial complex, $\sigma \in X$ a simplex

$lk(\sigma) =$ link of σ

$= \{ \tau \in X \mid \sigma \cup \tau \in X,\ \tau \cap \sigma = \emptyset \}$



Picture: $lk(\infty) = \mathbb{Z}$



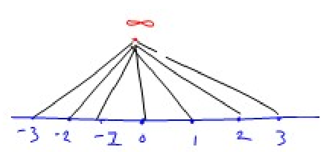
Notice: $\mathrm{Aut}(\mathbb{Z}) \cong \mathbb{Z} \rtimes \mathbb{Z}_2$

[infinite dihedral gp.]

WTS $\phi = Id \in \mathrm{Aut}(\mathcal{F})$.

$\boxed{\text{Step 4a}}$ $\phi|_{\infty \cup lk(\infty)} = Id$

Picture:

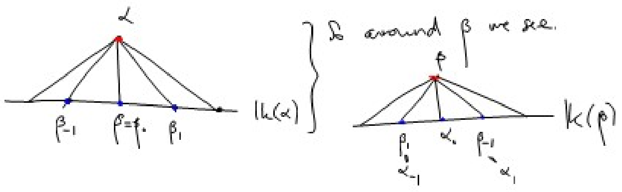


Since $\phi \in \mathrm{Aut}(\mathcal{F})$, $\phi(\infty) = \infty \Rightarrow \phi|_{lk(\infty)}$ is in $\mathrm{Aut}(\mathbb{Z})$. $\therefore \left. \begin{array}{l} \phi(0) = 0 \\ \phi(-1) = -1 \end{array} \right\} \Rightarrow \phi|_{lk(\infty)} = Id.$

$\boxed{\text{Step 4b}}$ If $\{\alpha, \beta\} \in \mathcal{F}$ and

$\phi|_{\alpha \cup lk(\alpha)} = Id$ <u>then</u> $\phi|_{\beta \cup lk(\beta)} = Id$

[<u>Crawling</u>] <u>Pf of This</u>: by picture:



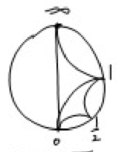
<u>So</u>: ϕ fixes β b/c $\beta \in lk(\alpha)$.

$\Rightarrow \phi$ preserves $lk(\beta)$. But ϕ fixes α_0, α_{-1}

$\Rightarrow \phi|_{lk(\beta)} = Id$.

$\boxed{\text{Step 5}}$ By induction on $d_{\mathcal{F}}(\infty, p/q)$

$\phi(p/q) = (p/q)$. Use step 4b. //Lemma

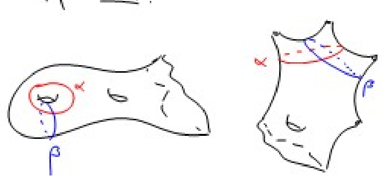


[Here $d_{\mathcal{F}}(\infty, p/q)$ is distance as measured in $\mathcal{F}^1$ the one-skeleton.]

<u>Def</u>: If α, β nonsep and $\iota(\alpha, \beta) = 1$ say α, β <u>dual</u>.

If α, β both <u>pants curves</u> and $\iota(\alpha, \beta) = 2$ say α, β <u>dual</u>.

<u>Pictures</u>:

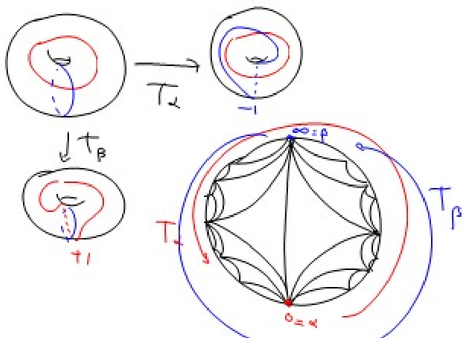


<u>Def</u>: $\alpha \subset S$ is a <u>pants curve</u> if α separating and S_α has a $S_{0,3}$ component.

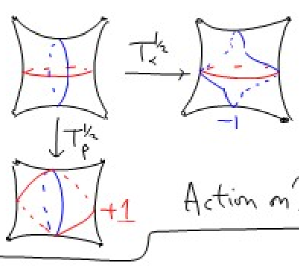
<u>Def</u>: for $S = S_{1,1}, S_1, S_{0,4}$ define $\mathcal{C}(S)$ to have vertex set $= \mathcal{S}(S)$ and simplices given by duality.

<u>Ex</u>: $\mathcal{C}(S) \cong \mathcal{F}$ in all three cases.

<u>Recall</u>:



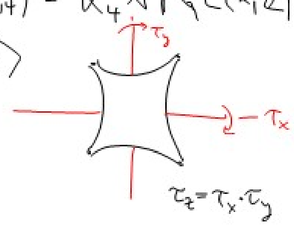
Left twists give counter-clockwise parabolics of $\mathcal{F}$



Action on $\mathcal{F}$ is identical.

<u>Lemma</u>: $\mathcal{MCG}(S_{0,4}) \cong K_4 \rtimes PGL(2, \mathbb{Z})$

[$K_4 = \langle \tau_x, \tau_y, \tau_z \rangle$

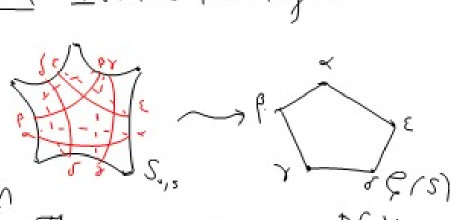


]

<u>Pf</u>: <u>Exercise</u> [Hint: Look for short exact sequence that splits.]

<u>Next Time</u>: <u>I</u>vanov's Thm in genus Zero.

<u>Def</u>:



<u>Pentagon</u>

<u>Lemma</u>: The pentagon is <u>unique</u>. Pf <u>Next time</u>